Objectives in conflict:

trade off methods and penalty functions

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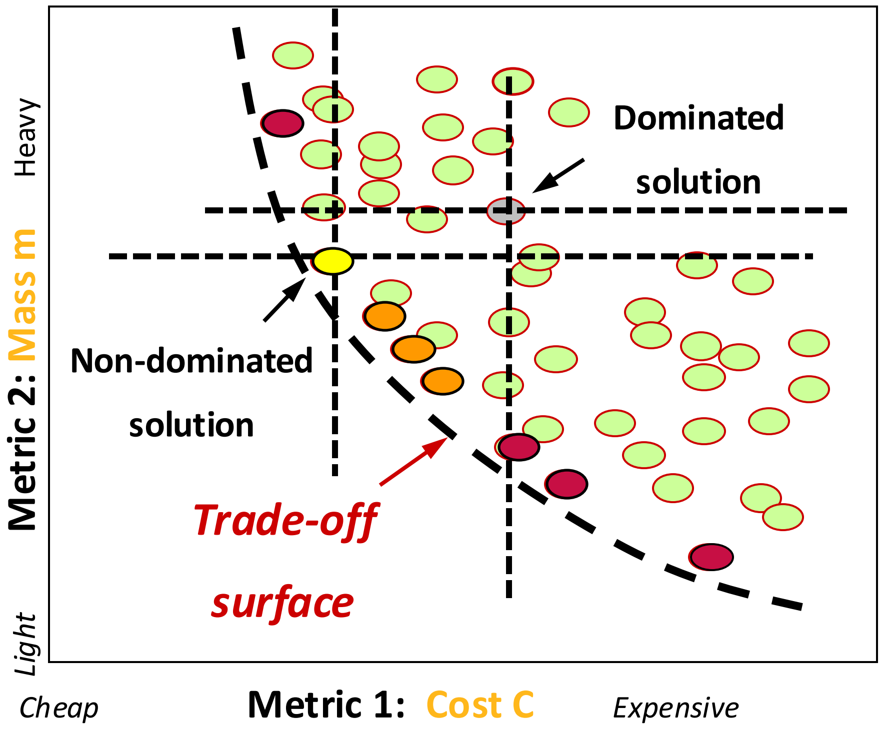


Learning objectives for this lecture unit

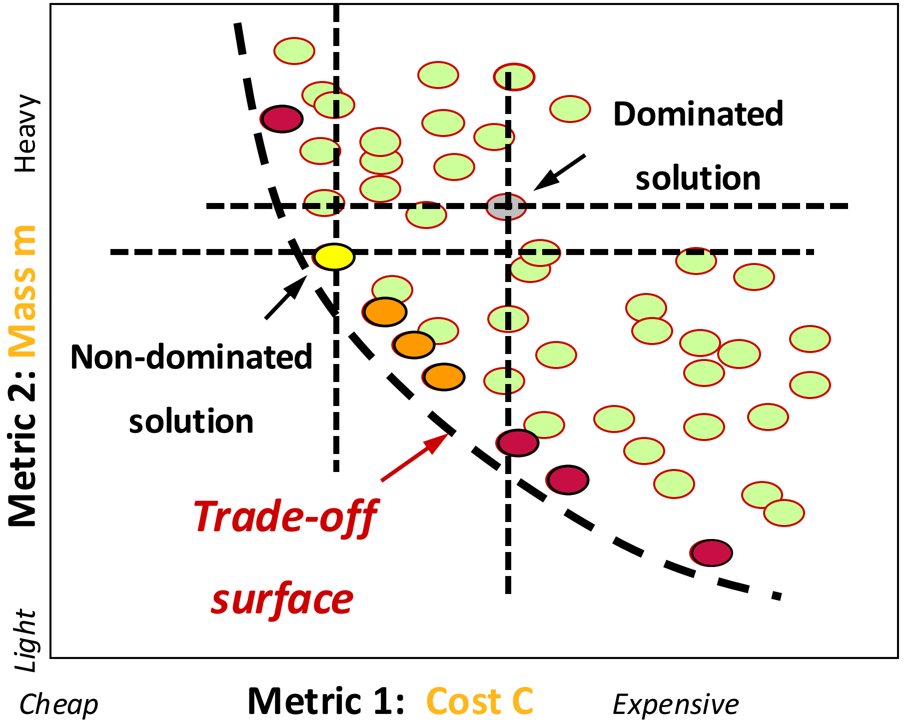
Ansys software mentioned	• Ansys Granta EduPack™, a teaching software for materials education

Intended Learning Outcomes	
Knowledge and Understanding	Knowledge on graphical trade-off methods and penalty functions
Skills and Abilities	Ability to select systematically when design objectives conflict
Values and Attitudes	Appreciation of the value of compromise in engineering design

Resources

- Text: "**Materials Selection in Mechanical Design**", 5th Edition by M.F. Ashby, Butterworth Heinemann, Oxford, 2016. Chapters 8-9
- Text: "**Materials and the Environment**", 2nd Edition by M.F. Ashby, Butterworth-Heinemann, Oxford 2012, UK. Chapters 9-10

Outline of lecture unit



- Almost always **2+ objectives** – they conflict
- **Trade-off methods**
- **Penalty functions** and exchange constants
- Two-objective minimisation using the Ansys Granta EduPack software

The selection strategy: materials

Design requirements:
expressed as
Constraints *and*
Objectives

- Able to be molded
- Water and UV resistant
- Stiff enough
- Strong enough
- *As cheap as possible*
- *(As light as possible)*

Comparison engine

- Screening
- Ranking
- Documentation

Data:
Material attributes
Process attributes
Documentation

- Density
- Price
- Modulus
- Strength
- Durability
- Process compatibility
- More…….

Multiple constraints and objectives

Design requirements set **constraints** – criteria for screening

objectives – criteria for optimising

Typical constraints

The material must be

- Electrically conducting
- Optically transparent…..

And meet target values of

- Stiffness
- Strength…..

And be able to be

- Die cast
- Welded ……

Dealing with multiple constraints is straightforward

Typical objectives

Minimize

- **Mass m** *(satellite components)*
- **Volume** *(mobile phones)*
- **Energy consumption** *(fridges)*
- **Carbon footprint** *(cars)*
- **Embodied energy** *(materials)*
- **Cost C** *(everything)*

Dealing with multiple objectives needs **trade-off methods**

Take, as example, simultaneously minimizing **mass m** and **cost C**

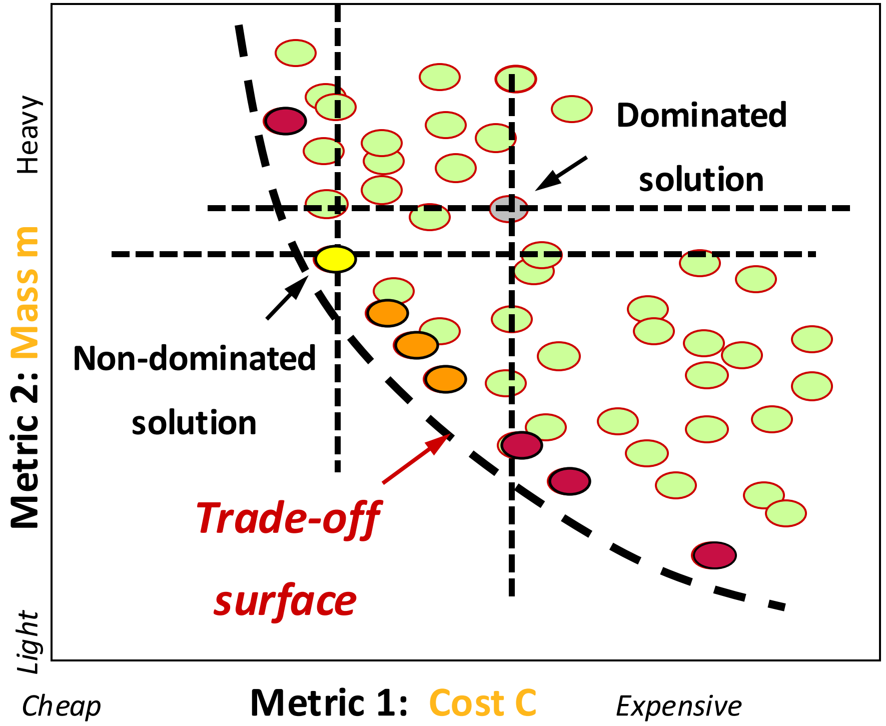
Multi-objective optimization: *the words*

- ***"Solution":*** a candidate that meets the constraints, but not necessarily optimum by either objective

- Plot solutions.
(***Convention***: express objectives to be ***minimized***)

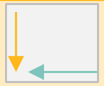
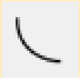
- ***"Dominated solution":*** one that is definitely non-optimal

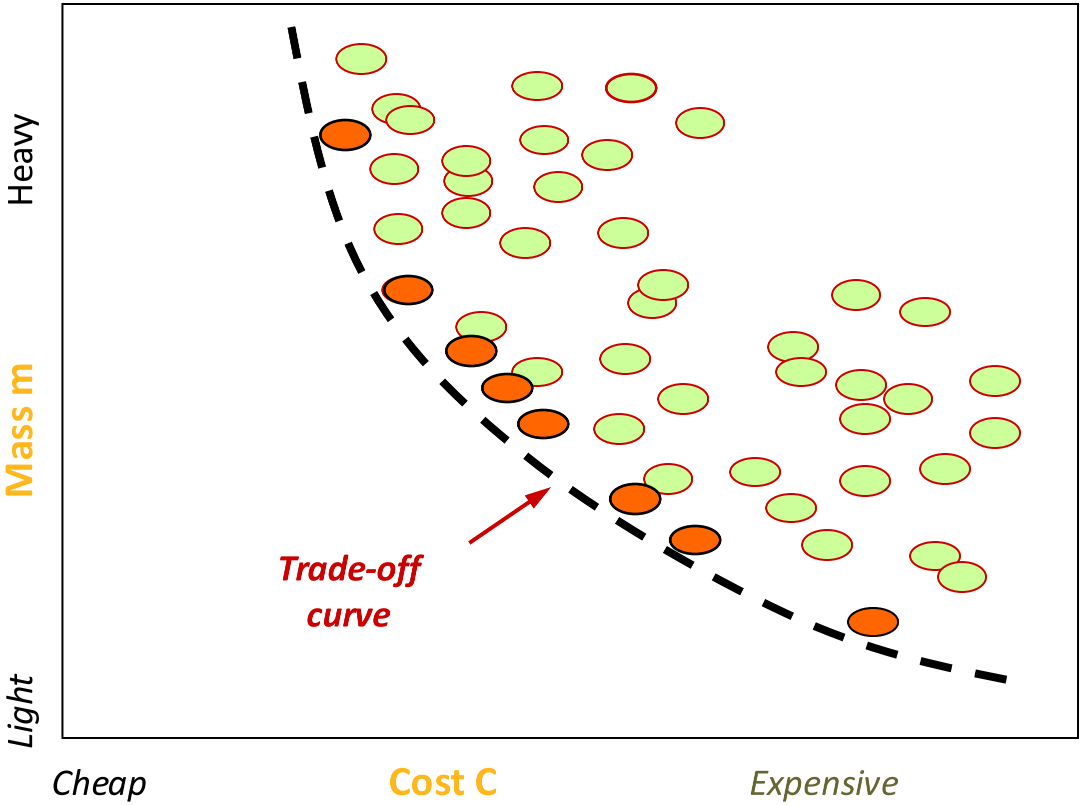
- ***"Non-dominated solution":*** one that is optimal by one metric (but not usually by both)



- ***"Trade-off surface":*** the surface on which the non-dominated solutions lie (Pareto Front). In our case a 2-dimensional curve

Finding a compromise: strategy 1

- **Make a trade-off plot** 
- **Sketch a trade-off curve** 
- **Use intuition** to select a solution on the trade-off curve



- "Solutions" nearest the surface offer the best **compromise** between mass and cost
- 8 out of 50
- Choose from among these - depends on how highly you value light weight

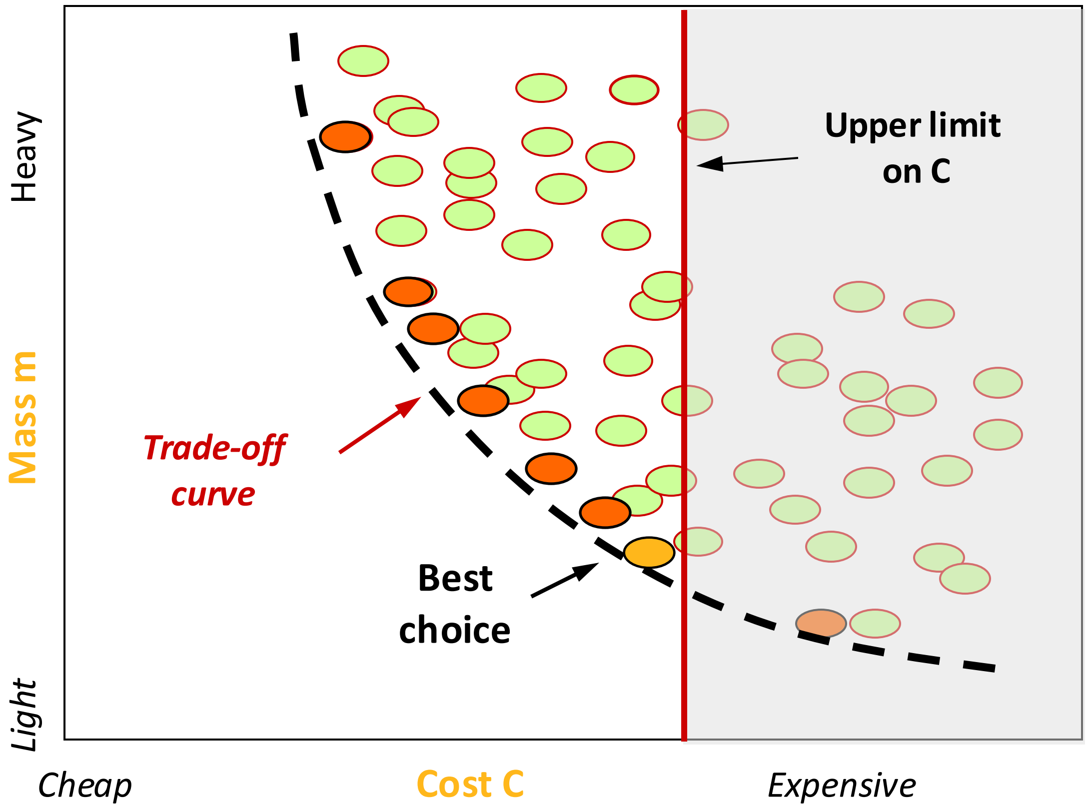
Finding a compromise: strategy 2

- ***Reformulate*** all but one of the objectives as constraints, setting an upper limit for it

OK **if budget limit**

- **BUT….cheating**

Cost is treated as *constraint*, not *objective*.



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Finding a compromise: strategy 3

Define locally-linear
Penalty function Z

$$Z = C + \alpha\, m$$

Seek solution with smallest Z

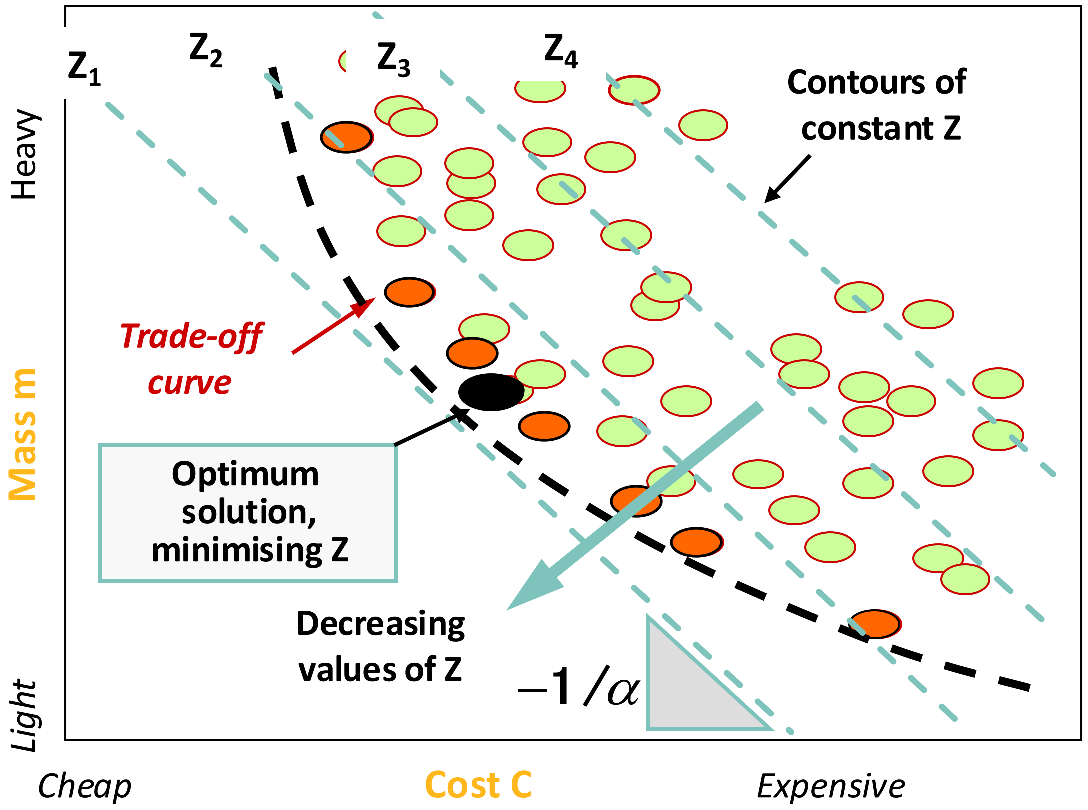
- Make **trade-off plot**

Plot on it contours of Z

$$m = -\frac{1}{\alpha} C + \frac{1}{\alpha} Z$$

Lines of Z have slope -1/α
(needs linear scales)

- **Read off** solution with lowest Z

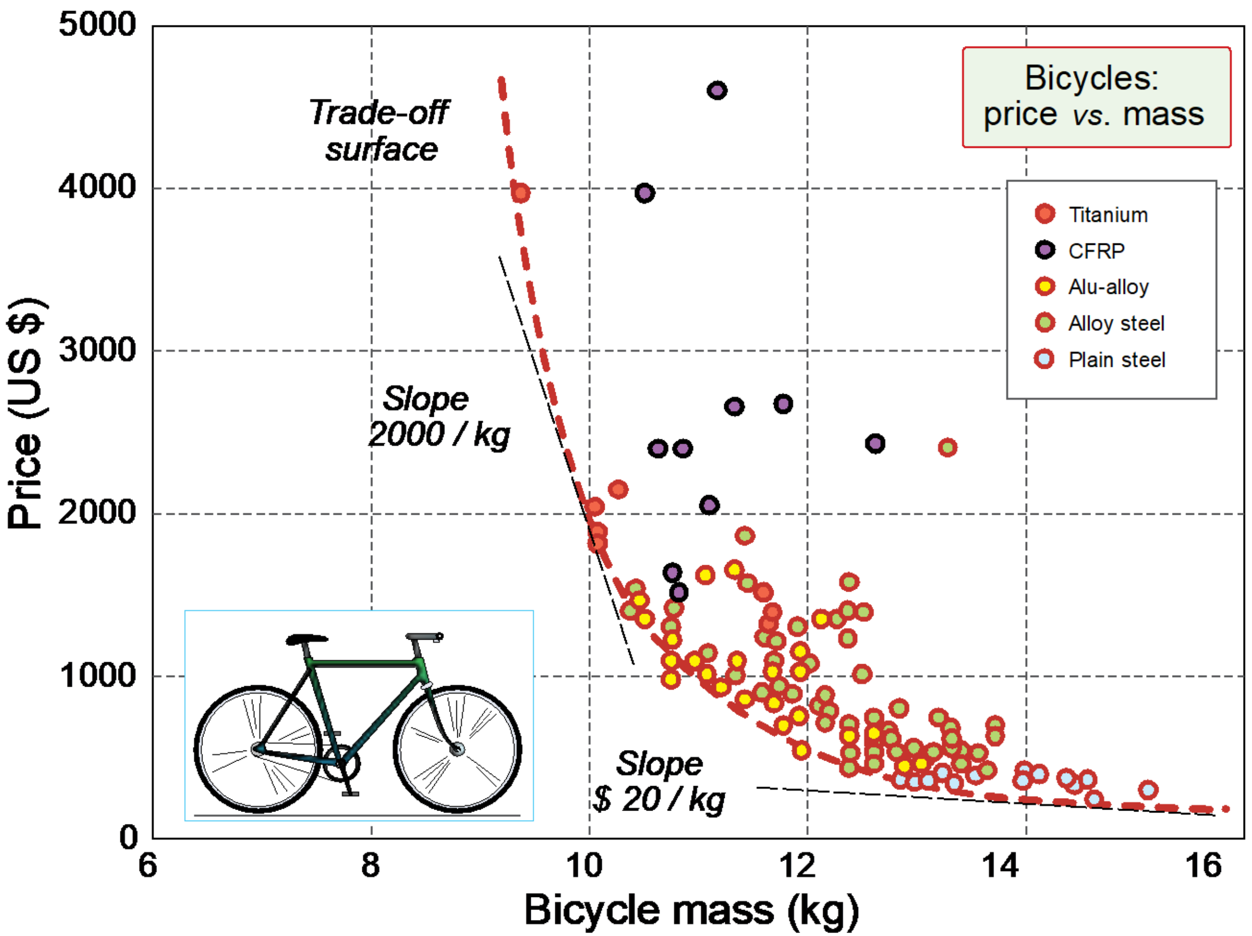


- **Two issues:**

(Q1) What is the so called exchange constant, α **?**

(Q2) What if we have ***Log***, not ***Linear*** scales?

(Q1) Example of graphical solution for teaching



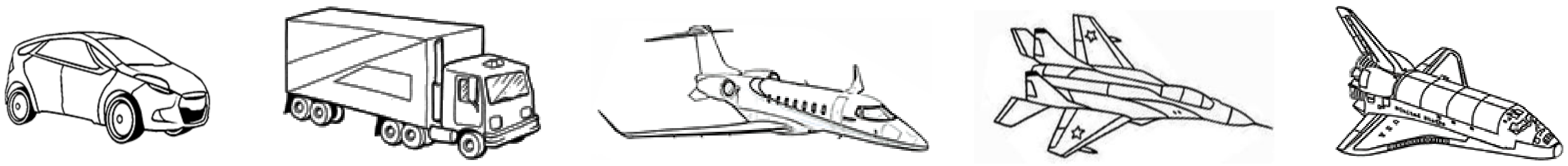
α determines a location on the trade-off curve and reflects priorities (price per kilo)

(Q1) Example: materials for transport systems

Choice of material depends on system

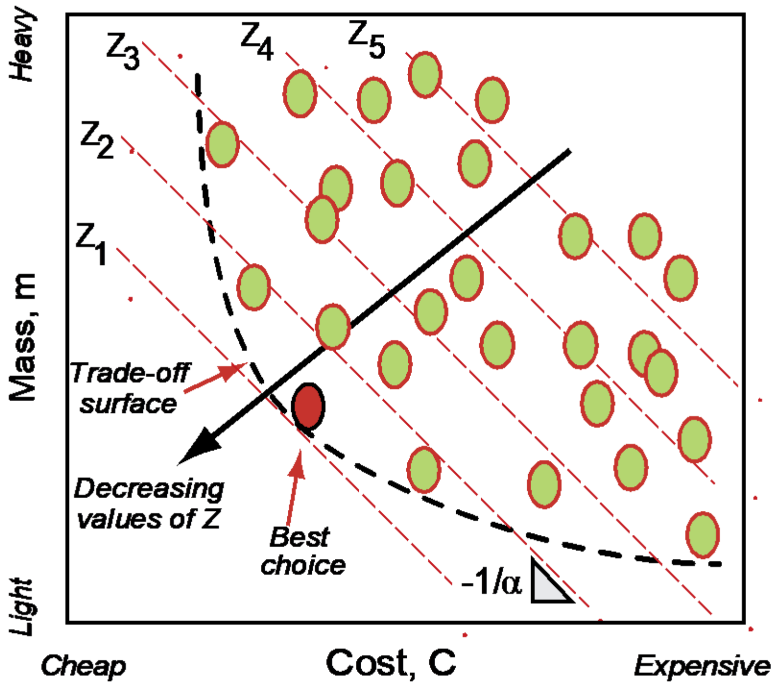
- **Mass**, in transport systems, means **fuel**
- **Life cost** = Initial cost, **C** + Fuel cost over life, scaling with mass **m**
- **Penalty function** $Z = \mathbf{C} + \alpha\, m$
- Must establish **exchange constant,** α

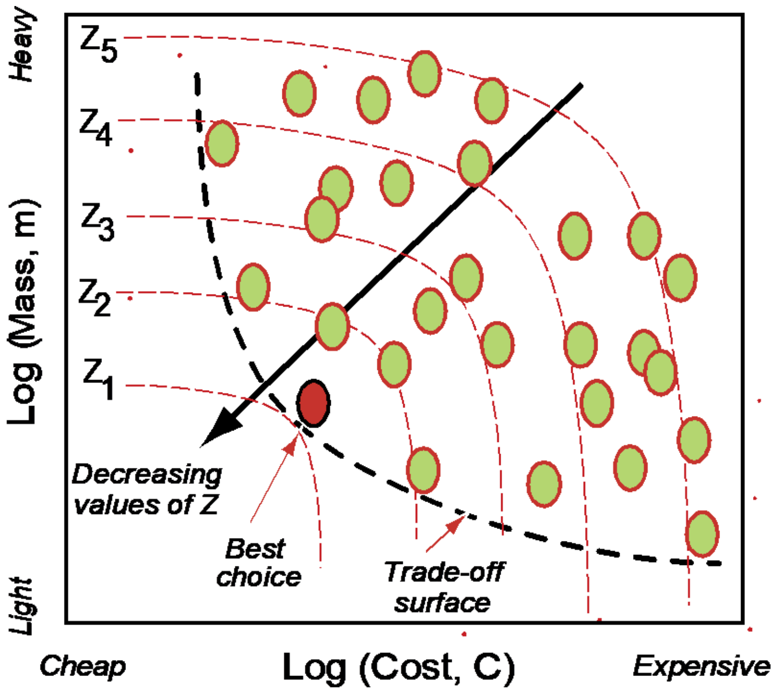
Units: $ (C), kg (m), $/kg (α)



	Steel	**Steel / Alu**	**Alu / (composite)**	**Alu / Ti / composites**	**Composites**
α ($/kg)	**3 – 6**	**6 – 20**	**100 – 600**	**600 – 2,000 (?)**	**5,000 – 10,000**

(Q2) Linear penalty functions go with linear axes



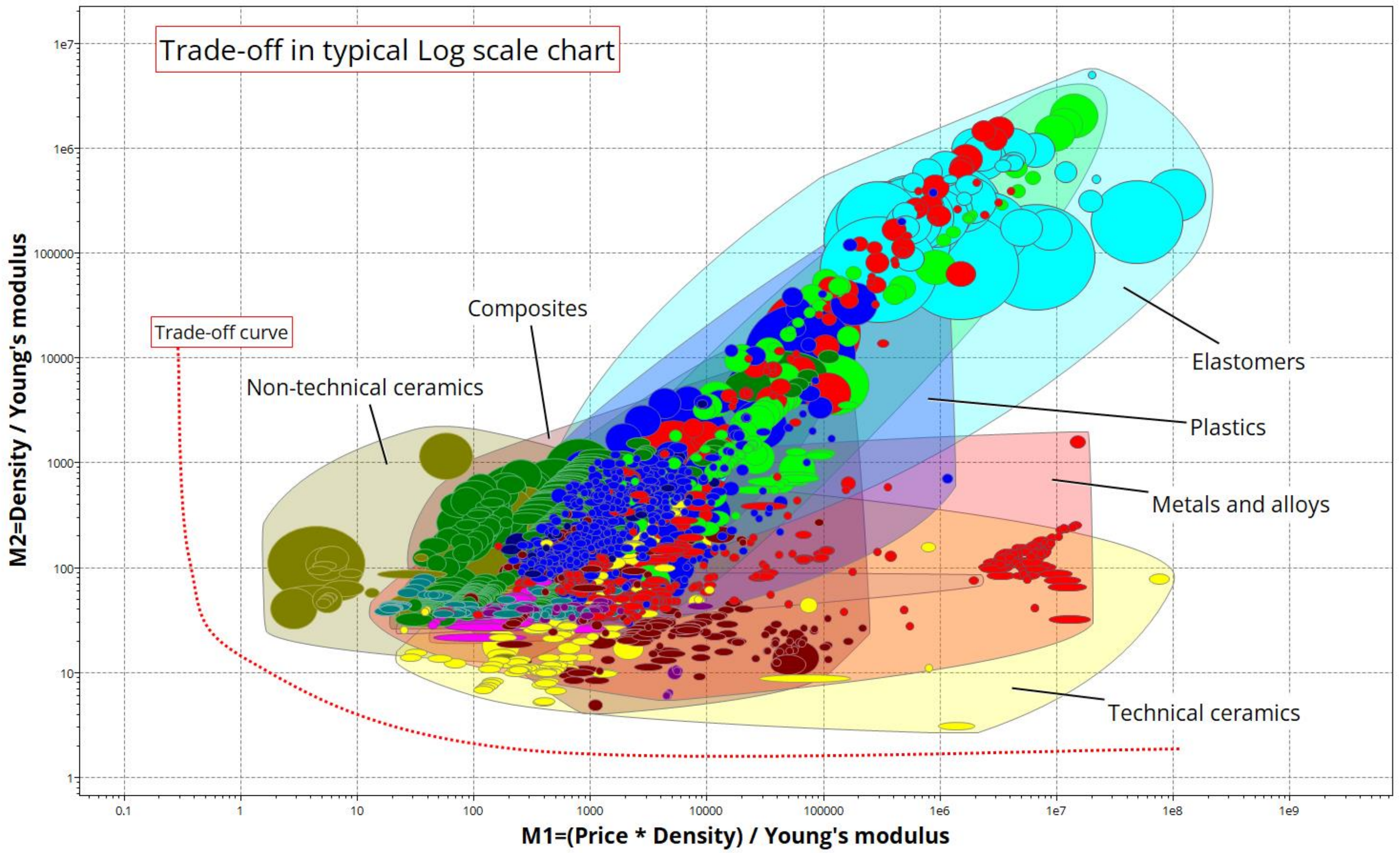


- Set your axes to linear before plotting property charts for linear penalty functions
- Logarithmic scales give the same best choice but Z no longer appears as straight

(Q2) Example of two-objective Log chart

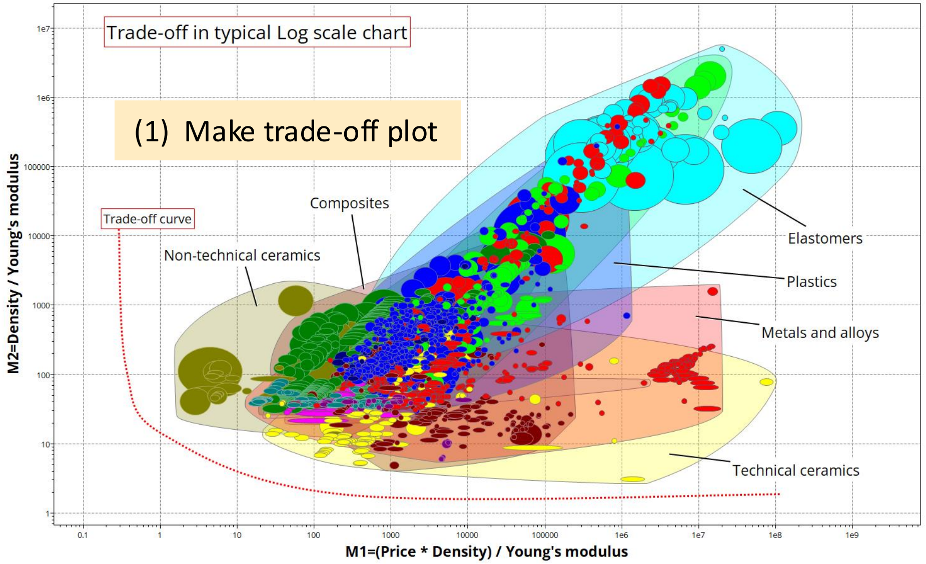
Minimum mass and cost for member in tensile or compressive load and stiffness-limited design: *Log* scale axes

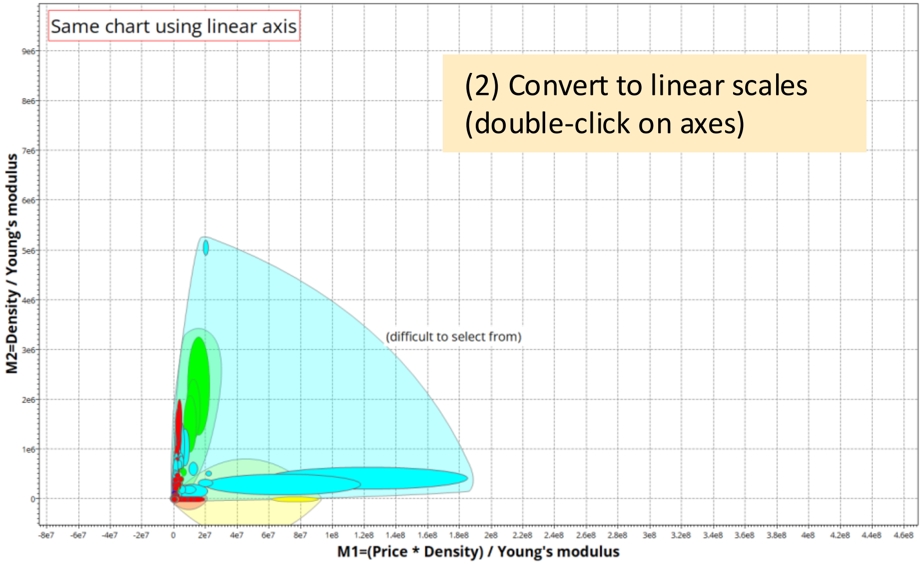
Minimize $M2 = \rho / E$

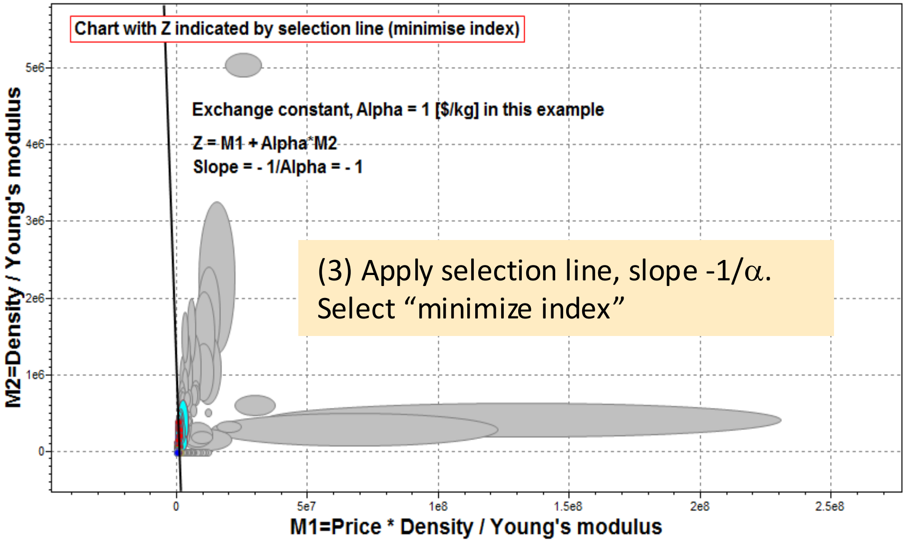


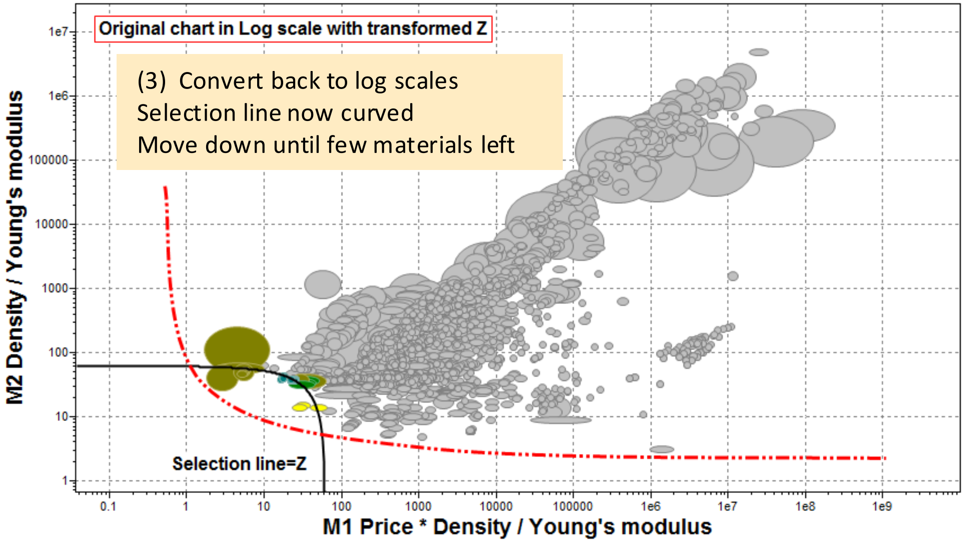
Minimize $M1 = C_m * \rho / E$

(Q2) How to use a penalty function in bubble charts









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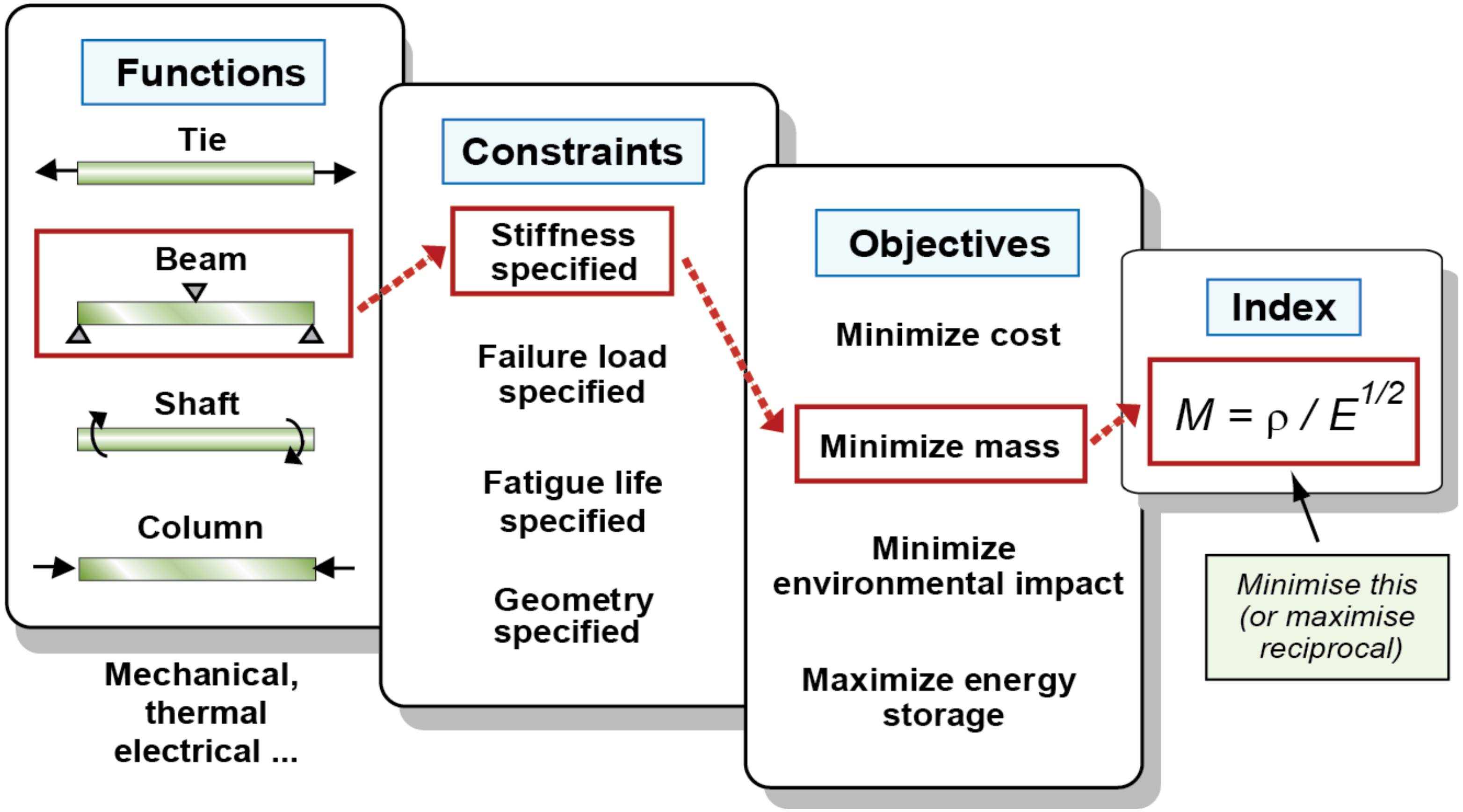
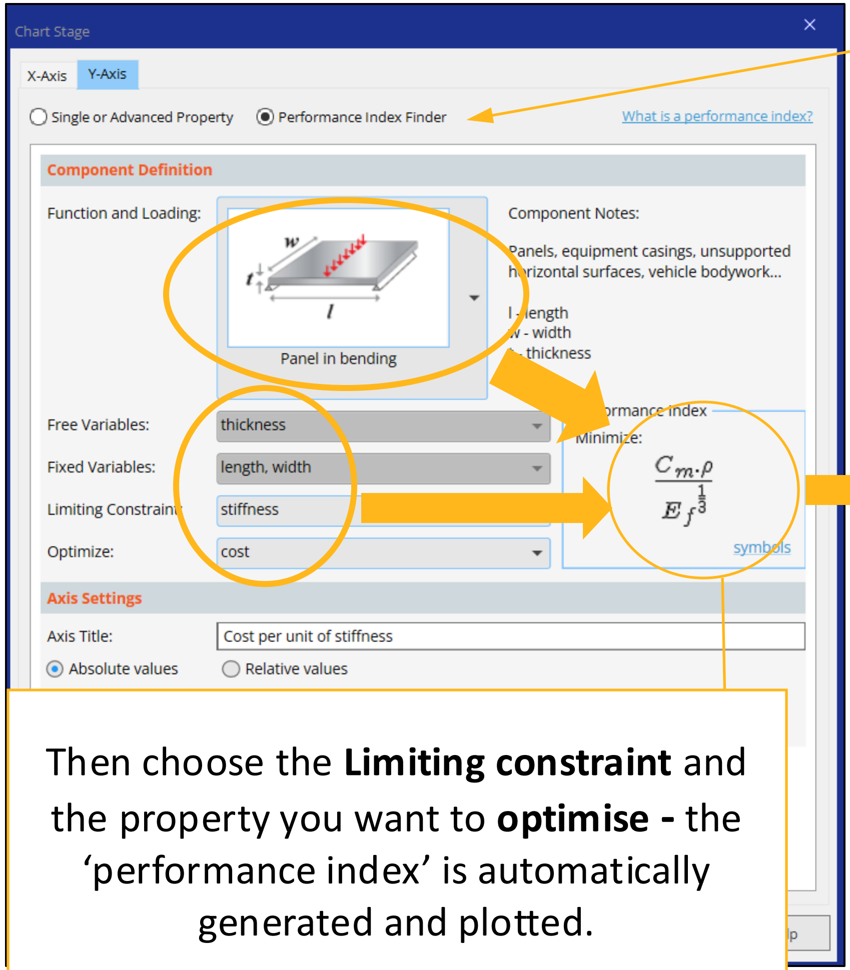
Performance Index finder methodology

A performance index is a group of material properties that limits the performance of a design



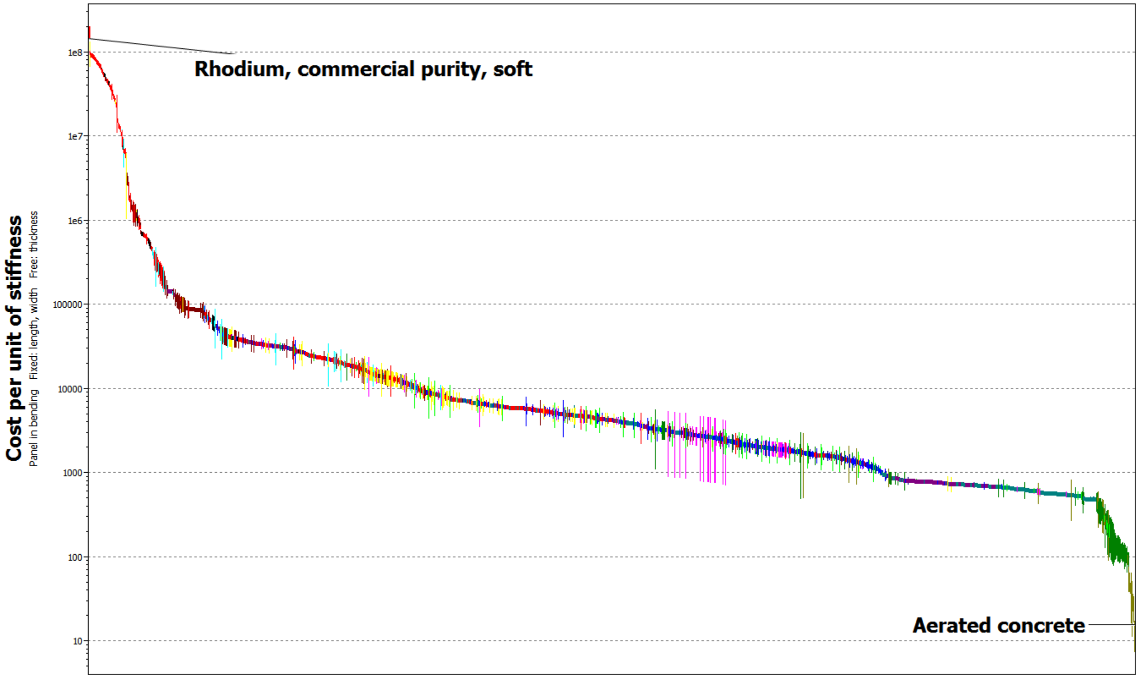
Performance Index finder

Chart/Index



To plot a performance index, click here and choose the model that best fits your function and loading.

Then choose the **Limiting constraint** and the property you want to **optimise -** the 'performance index' is automatically generated and plotted.



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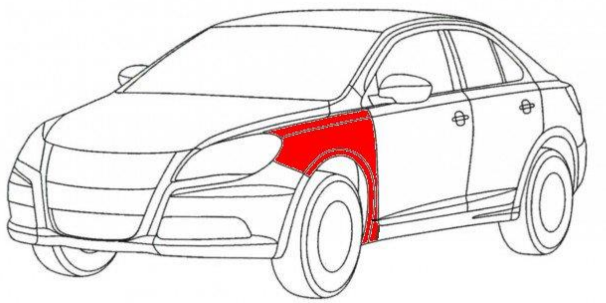
Example: trade-off between cost and weight

- The scenario:
  - Select a material for an exterior panel of a vehicle
  - It must be as light and cheap as possible
  - Stiffness is the most important constraint



Define locally-linear
Penalty function Z

$$Z = \alpha\, m + C$$

Seek solution with smallest Z (1)

$$C = \frac{C_m \rho}{E^{1/3}}$$

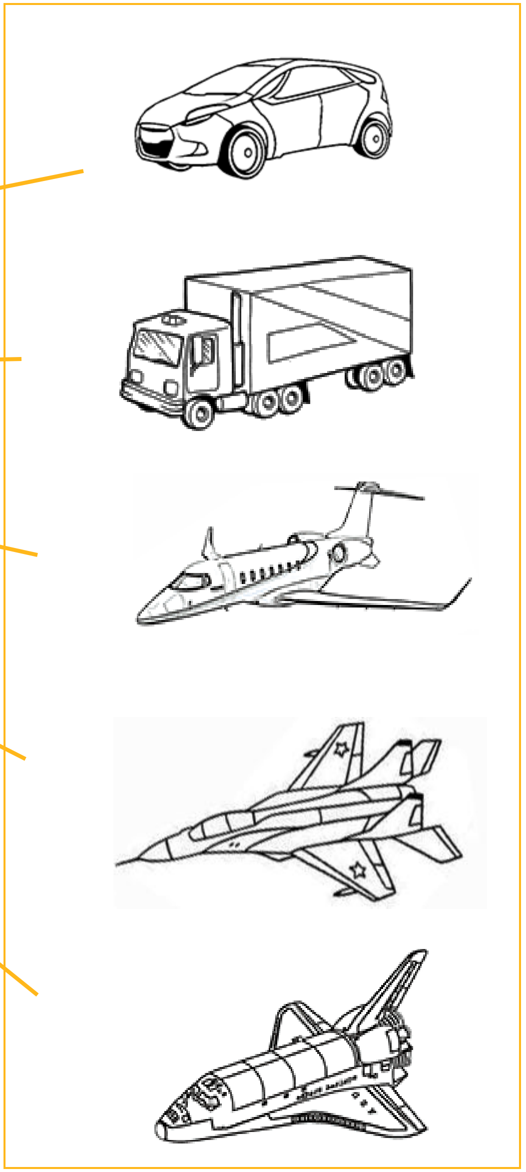
$$m = \frac{\rho}{E^{1/3}}$$

- α: how much are you willing to pay for each extra kg ***(exchange constant)***?

Granta EduPack software

The exchange constant α for transport

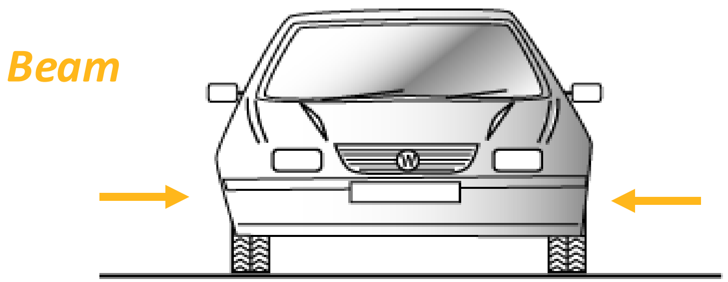
Transport system	α ($ per kg)
Family car	3 to 6
Truck	5 to 20
Civil aircraft	100 to 500
Military hardware	500 to 2000
Space vehicle	3000 to 10,000



How to get values of α?

- **Full life costing: fuel saving, extra payload**
- **Analysis of historic data;**
- **Interviews, surveys**

Two ways to find materials for auto bumpers



Function — *Absorb impact, transmit load to energy-absorbing units or supports*

Objectives — *Minimize mass and material cost*

Criteria — ***Mass m** per unit bending strength* | ***Cost C** per unit bending strength*

Beam in bending Index to minimize:

$$m = \frac{\rho}{\sigma_y^{2/3}} \qquad C = \frac{C_m \rho}{\sigma_y^{2/3}}$$

- C_m = Material cost / kg
- ρ = Density, kg/m^3
- σ_y = Yield strength, MPa
- α = exchange constant , \$/kg

Penalty function

$$Z = C + \alpha m = \frac{\rho}{\sigma_y^{2/3}}\left(C_m + \alpha\right)$$

Bar chart selection using the penalty function

$$Z = \frac{\rho}{\sigma_y^{2/3}}\left(C_m + \alpha\right)$$

*Use the "**Advanced**" facility to make the penalty function*

(Density / (Yield strength^0.66))

***(Price + 10)**

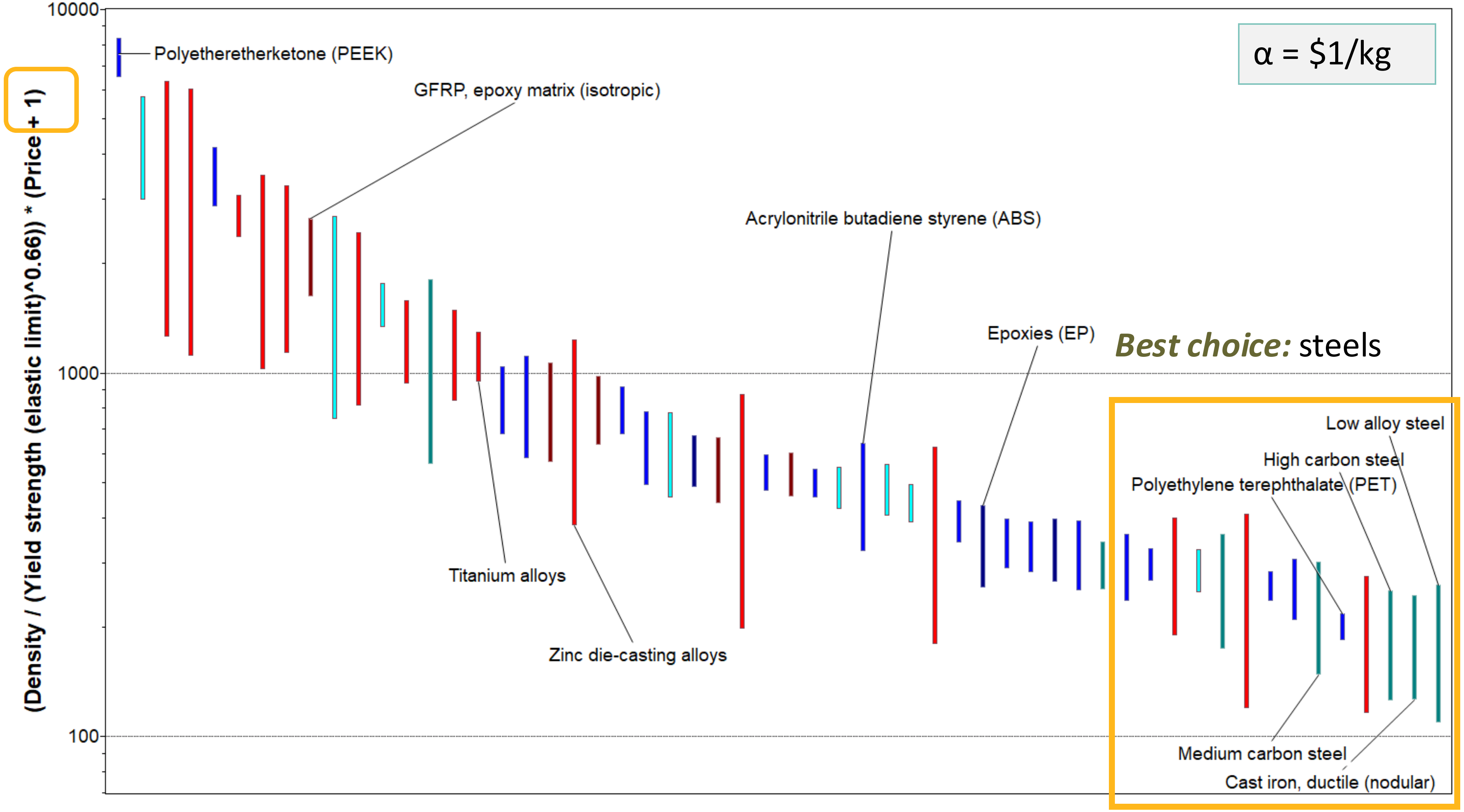
The value of the exchange constant

+ **-** **/** ***** **^** **(** **)**

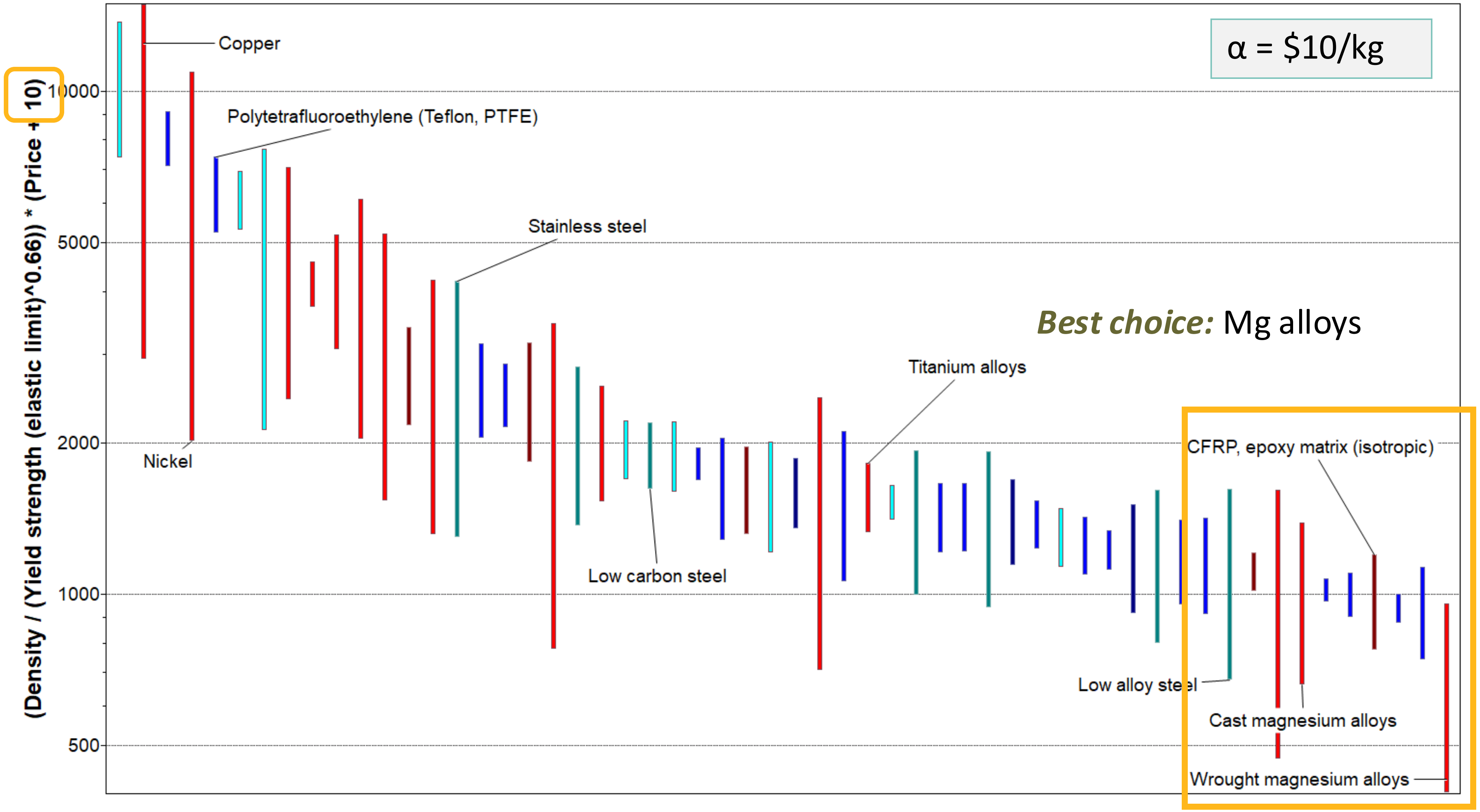
List of properties

- Density
- Price
- Tensile strength
- etc

Bar chart selection using the penalty function

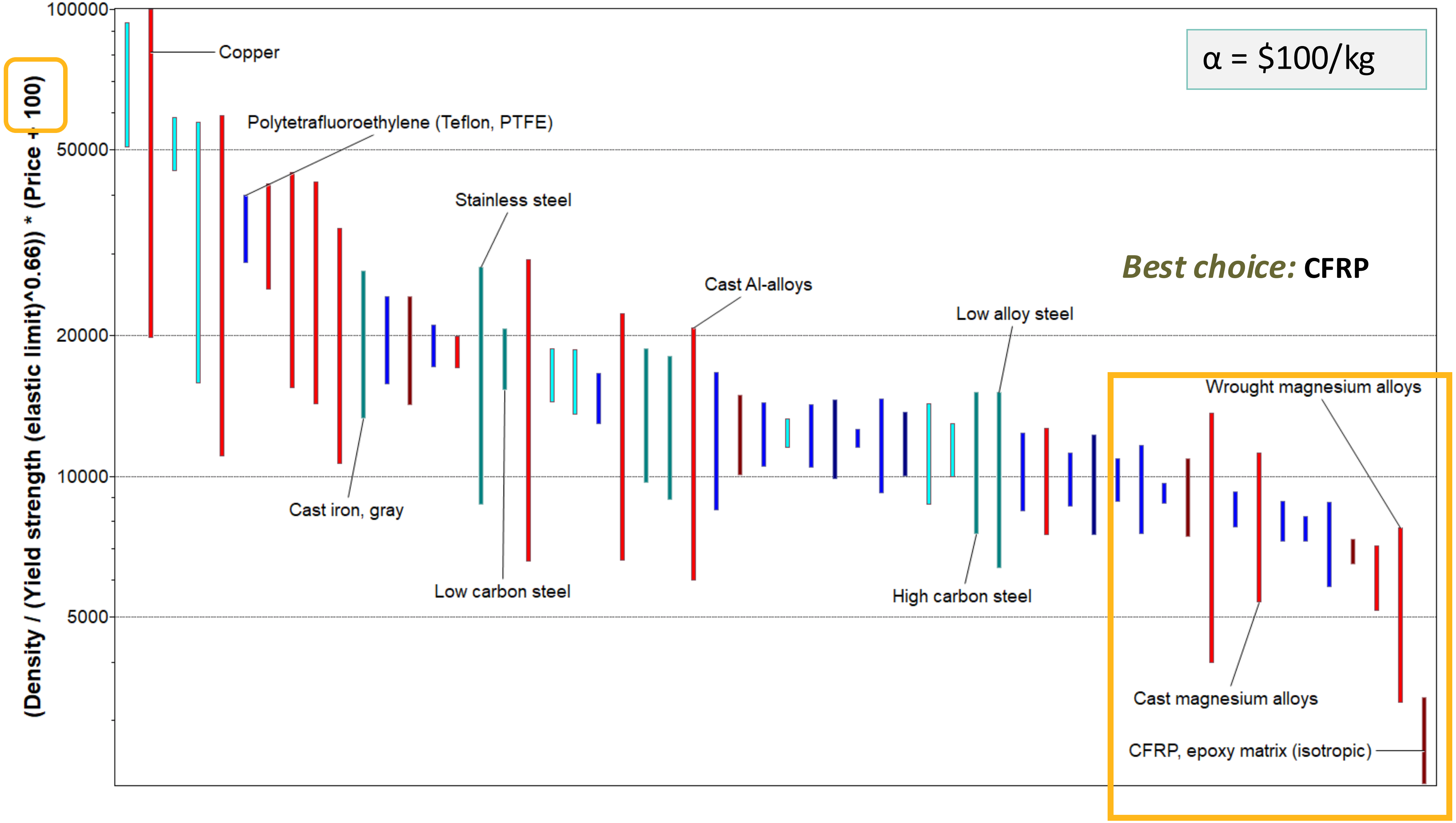


Bar chart selection using the penalty function



Best choice: Mg alloys

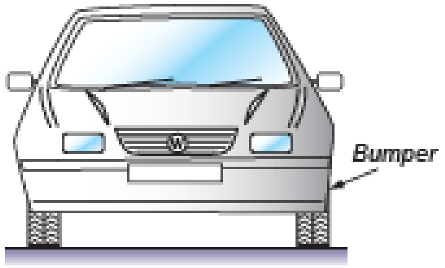
Bar chart selection using the penalty function



Best choice: **CFRP**

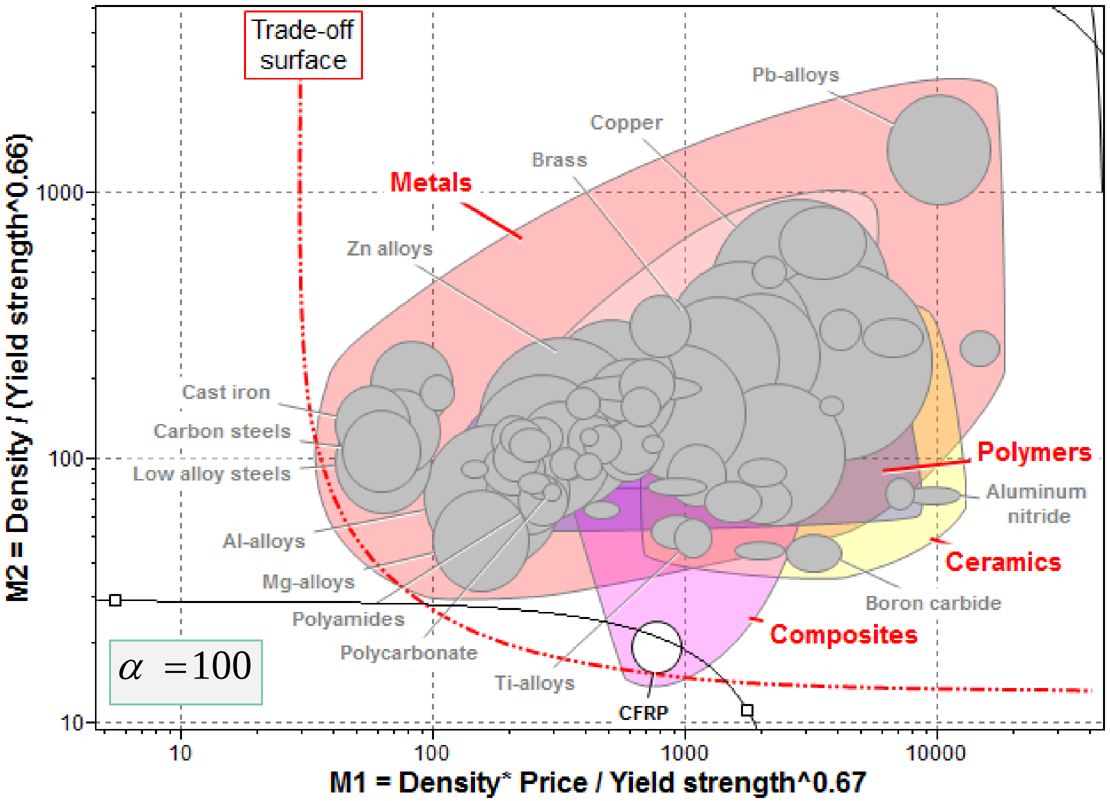
Bubble chart selection using penalty function

Strong bumper, minimum weight and cost



Minimize weight ⇨ $M_2 = \frac{\rho}{\sigma_y^{2/3}}$

Minimize cost ⇨ $M_1 = \frac{C_m \rho}{\sigma_y^{2/3}}$



Penalty function $Z = M_1 + \alpha M_2$

α *= 1 $/kg* ⇨ *Low alloy steels, Carbon steels,*

α *= 10 $/kg* ⇨ *Aluminum alloys, Magnesium alloys*

α *= 100 $/kg* ⇨ *Carbon-fiber reinforced composites*

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Summary

- Real design involves conflicting objectives –

 often ***technical performance*** vs. ***economic performance*** (cost).

- **Trade-off plots** reveal options

- If the **exchange constant** is known –

penalty function allows unambiguous choice

- The **penalty function** technique can be applied to bar charts or bubble charts in the Ansys Granta EduPack software for interactive and visual selection

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