Material and shape

Materials for efficient structures

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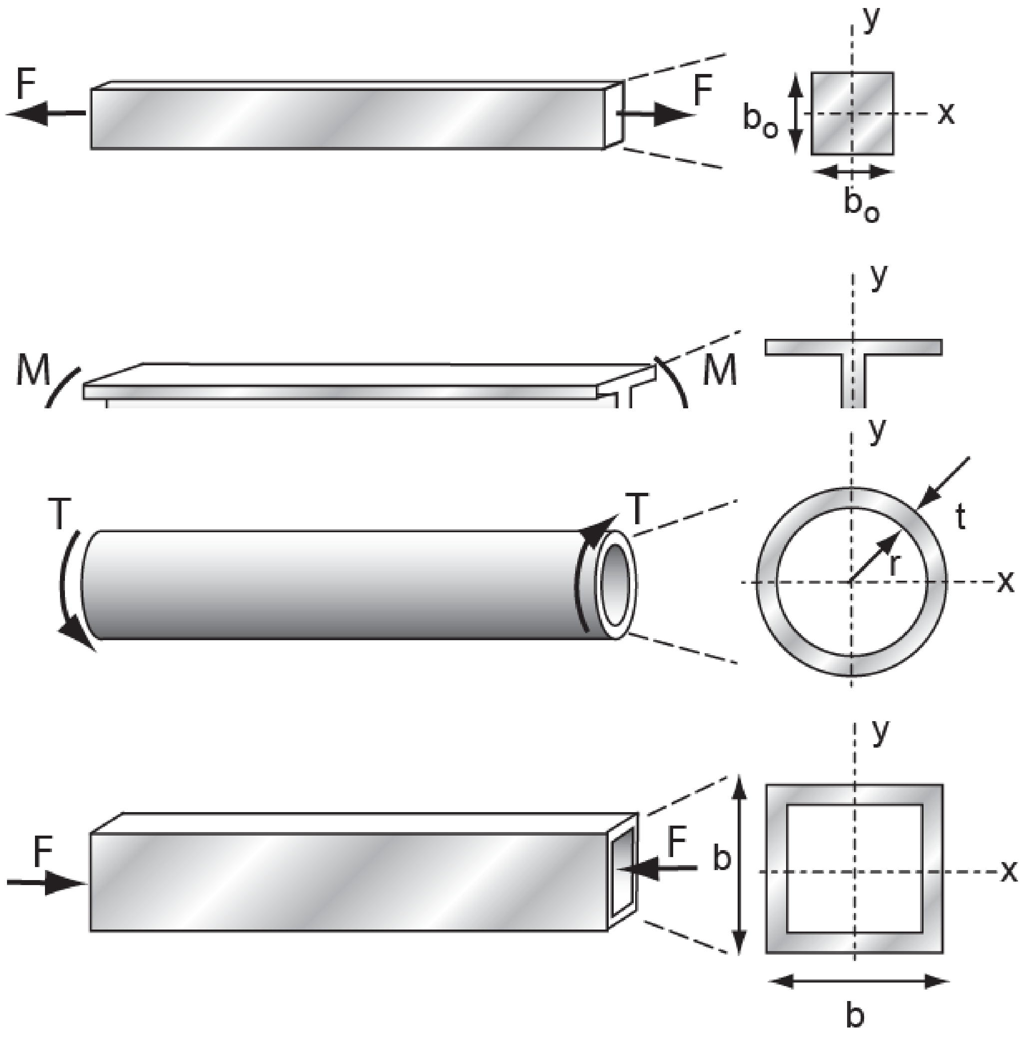


Learning objectives for this lecture unit

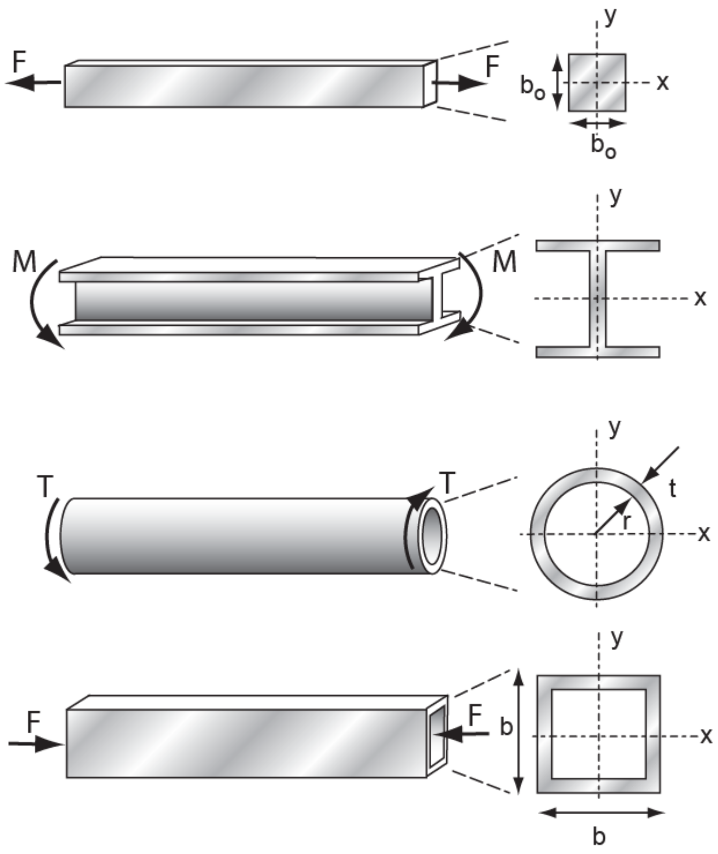
Ansys software mentioned	• Ansys Granta EduPack™, a teaching software for materials education

Intended Learning Outcomes	
Knowledge and Understanding	Understanding of the concept of shape efficiency Understanding of how to explore and optimize mechanical design structures
Skills and Abilities	Ability to select efficient material-shape combinations Ability to select structural sections for prescribed design requirements
Values and Attitudes	Awareness of how materials and shape interact Appreciation of how shape and material properties interact

Resources

- **Text:** "Materials Selection in Mechanical Design", 5th edition by M.F. Ashby, Butterworth Heinemann, Oxford, 2016, Chapters 10-11
- **Software:** The Ansys Granta EduPack software Structural sections data-table and the Built Environment Level 2 database

Outline

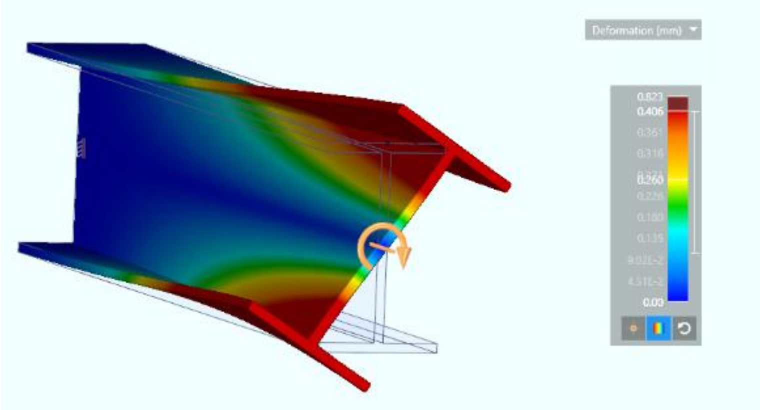


- Efficient shapes: tubes, I-beams etc
- The shape factor and shape limits
- Material indices that include shape
- Graphical ways of dealing with shape
- The Ansys Granta EduPack software Structural Sections data-table

Shape efficiency

- When materials are loaded in bending, in torsion, or are used as slender columns, section shape becomes important

- ”**Shape**” = cross section formed to a
  - tubes
  - I-sections
  - tubes
  - hollow box-section
  - sandwich panels
  - ribbed panels



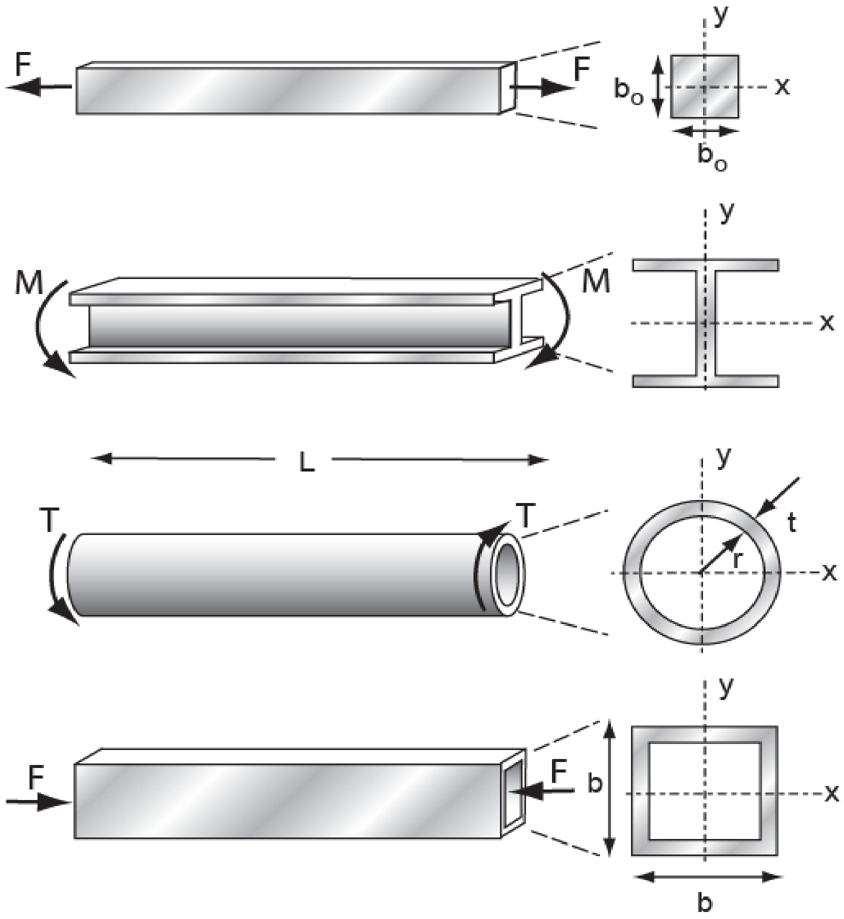
- “**Efficient**” = use least material for given stiffness or strength

- Shapes to which a material can be formed are limited by the material itself

- Goals: understand the limits to shape develop methods for co-selecting material and shape

Shape and mode of loading

Standard structural members



Tie — Area A matters, not shape

Beam — Area A and shape I_{XX}, I_{YY} matter

Torsion bar — Area A and shape J matter

Column — Area A and shape I_{min} matter

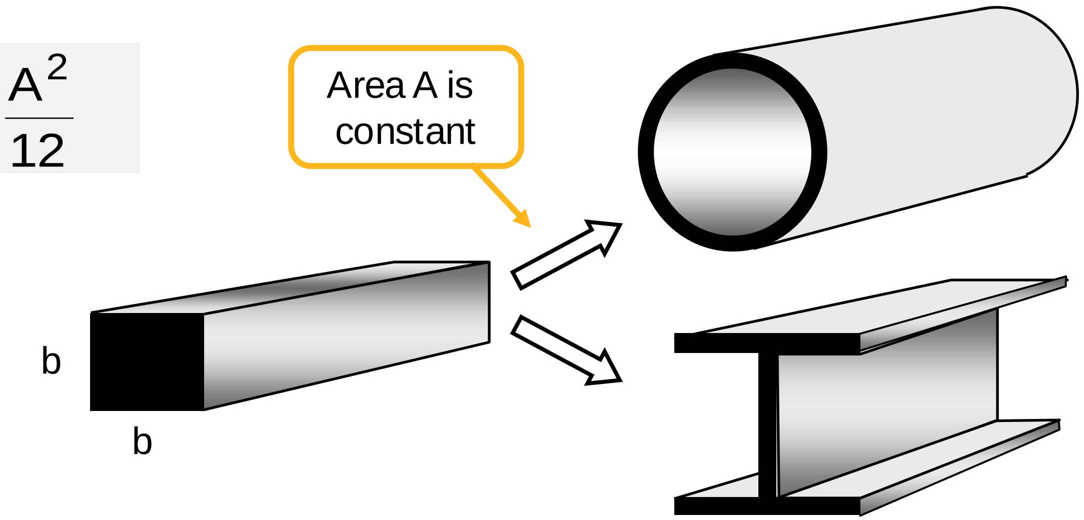
Certain materials can be made to certain shapes: what is the best combination?

Shape efficiency: bending stiffness

- Take ratio of bending stiffness S of shaped section to that (S_o) of a neutral reference section of the same cross-section area
- Define a standard reference section: a solid square with area $A = b^2$
- Second moment of area is **I**; stiffness scales as E**I.**

$$I_o = \frac{b^4}{12} = \frac{A^2}{12}$$

Area $A = b^2$



$$I = \int y^2\, b(y)\, dy$$

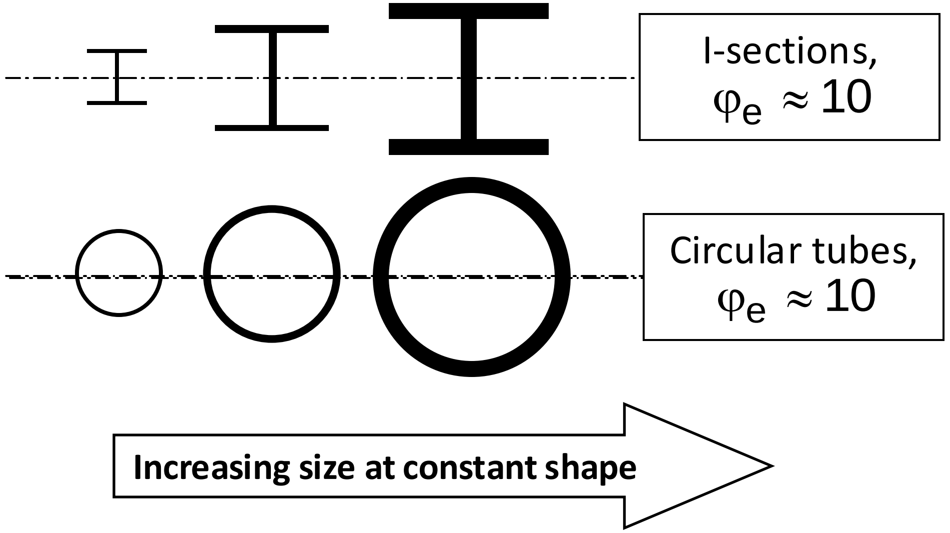
Area A and modulus E unchanged

Define **shape factor for elastic bending,** measuring efficiency, as

$$\boxed{\varphi_e = \frac{S}{S_o} = \frac{EI}{EI_o} = 12\frac{I}{A^2}}$$

Properties of the shape factor

- The shape factor is dimensionless – a pure number.
- It characterizes shape.



I-sections,
$\varphi_e \approx 10$

Circular tubes,
$\varphi_e \approx 10$

Increasing size at constant shape

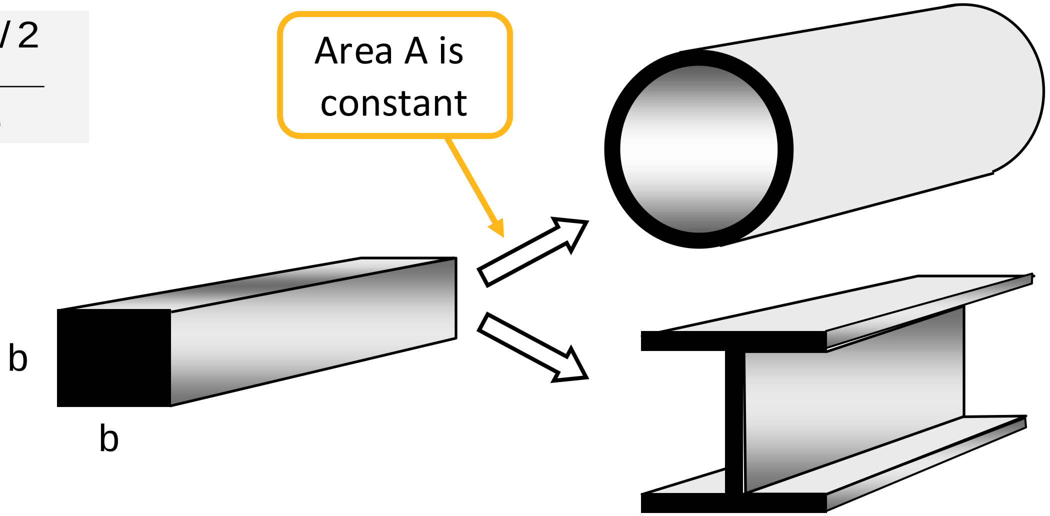
- Each of these is roughly 10 times stiffer in bending than a solid square section of the same cross-sectional area

Shape efficiency: bending strength

- Take ratio of bending strength F_f of shaped section to that ($F_{f,o}$) of a neutral reference section of the same cross-section area
- Section modulus of area is Z; strength scales as $\sigma_y Z$

$$Z_o = \frac{b^3}{6} = \frac{A^{3/2}}{6}$$

Area A = b^2



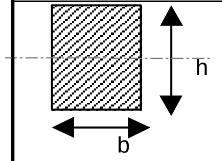
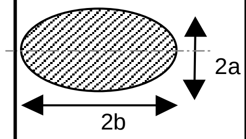
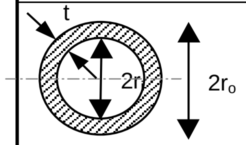
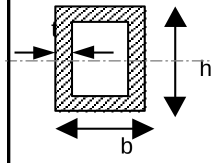
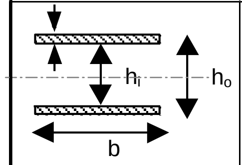
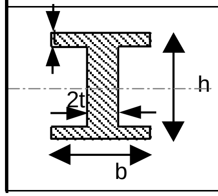
$$Z = \frac{I}{y_{max}}$$

Area A and yield strength σ_y unchanged

Define **shape factor for onset of plasticity (failure),** measuring efficiency, as

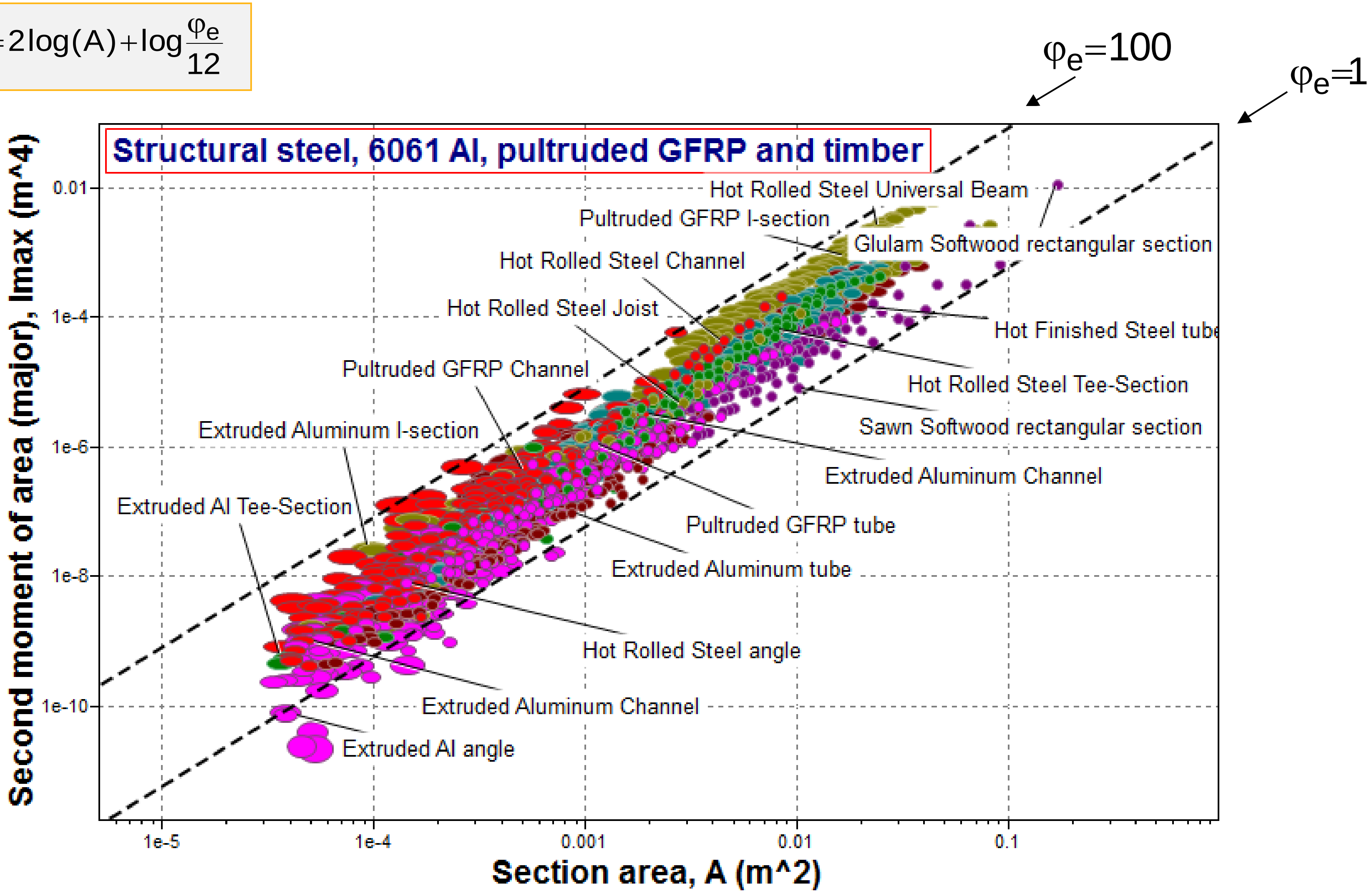
$$\varphi_f = \frac{F_f}{F_{fo}} = \frac{\sigma_y Z}{\sigma_y Z_o} = 6\frac{Z}{A^{3/2}}$$

Tabulation of shape factors

Section shape	Area A m	Second moment I, m^4	Elastic shape factor
	bh	$\frac{bh^3}{12}$	$\frac{h}{b}$
	πab	$\frac{\pi}{4}a^3 b$	$\frac{3}{\pi}\frac{a}{b}$
	$\pi(r_o^2 - r_i^2)$ $\approx 2\pi r t$	$\frac{\pi}{4}(r_o^4 - r_i^4)$ $\approx \pi r^3 t$	$\frac{3}{\pi}\left(\frac{r}{t}\right)$ $(r >> t)$
	$2t(h+b)$ $(h, b >> t)$	$\frac{1}{6}h^3 t(1 + 3\frac{b}{h})$	$\frac{1}{2}\frac{h}{t}\frac{(1+3b/h)}{(1+b/h)^2}$ $(h, b >> t$
	$b(h_o - h_i)$ $\approx 2bt$ $(h, b >> t)$	$\frac{b}{12}(h_o^3 - h_i^3)$ $\approx \frac{1}{2}bth_o^2$	$\frac{3}{2}\frac{h_o^2}{bt}$ $(h, b >> t)$
	$2t(h+b)$ $(h, b >> t)$	$\frac{1}{6}h^3 t(1 + 3\frac{b}{h})$	$\frac{1}{2}\frac{h}{t}\frac{(1+3b/h)}{(1+b/h)^2}$ $(h, b >> t)$

What values of φ_e exist in reality?

$$\varphi_e = \frac{12\,I}{A^2} \Rightarrow \log(I) = 2\log(A) + \log\frac{\varphi_e}{12}$$



Limits for shape factors φ_e and φ_f

- There is an upper limit to shape factor for each material

Material	Max φ_e	Max φ_f
Steels	65	13
Aluminum alloys	44	10
GFRP and CFRP	39	9
Unreinforced polymers	12	5
Woods	8	3
Elastomers	<6	-
Other materials	...can calculate	

- Limit set by: (a) manufacturing constraints
  (b) local buckling

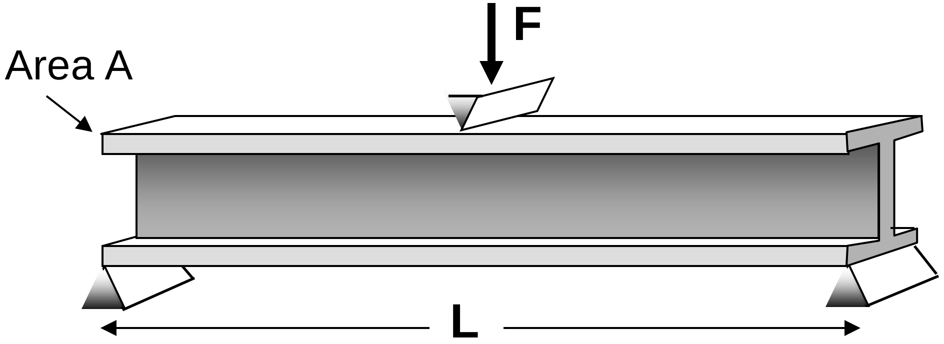
- Theoretical limit:

$$\boxed{\varphi_e \approx 2\sqrt{\frac{E}{\sigma_y}}}$$

Modulus

Yield strength

Indices that include shape



Function *Beam* (shaped section).

Constraint *Bending stiffness* = S:

$$S = \frac{CEI}{L^3}$$

I is the second moment of area:

$$\varphi_e = 12\frac{I}{A^2} \qquad A = \left(\frac{12I}{\varphi_e}\right)^{1/2}$$

Objective *Minimise mass*, m, where:

$$m = AL\rho$$

$$m = \left(\frac{12\,S\,L^5}{C}\right)^{1/2}\left(\frac{\rho}{(\varphi_e E)^{1/2}}\right)$$

m = mass
A = area
L = length
ρ = density
b = edge length
S = stiffness
I = second moment of area
E = Youngs Modulus

Chose materials with smallest $\left(\frac{\rho}{(\varphi_e E)^{1/2}}\right)$

Selecting material-shape combinations

- Materials for stiff, *shaped* beams of minimum weight
- Fixed shape (φ_e fixed): choose materials with low $\dfrac{\rho}{E^{1/2}}$
- Shape φ_e a variable: choose materials with low $\dfrac{\rho}{(\varphi_e E)^{1/2}}$

Material	ρ, Mg/m³	E, GPa	$\varphi_{e,max}$	$\rho / E^{1/2}$	$\rho / (\varphi_{e,max} E)^{1/2}$
1020 Steel	7.85	205	65	0.55	0.068
6061 T4 Al	2.70	70	44	0.32	0.049
GFRP	1.75	28	39	0.35	0.053
Wood (oak)	0.9	13	8	0.25	0.088

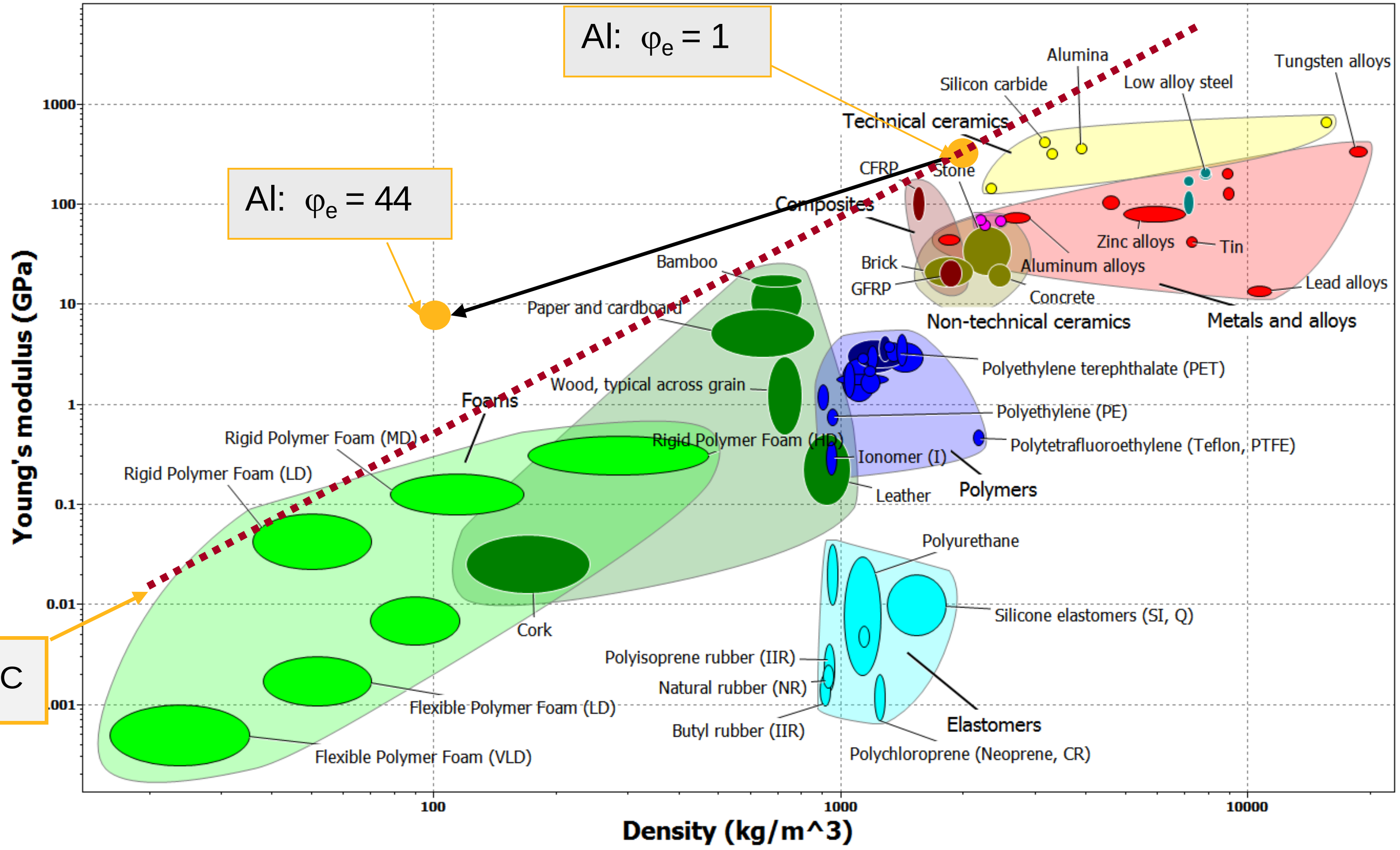
- Commentary: Fixed shape (up to $\varphi_e = 8$): wood is best

  Maximum shape ($\varphi_e = \varphi_{e,max}$): Al-alloy is best

  Steel recovers some performance through high $\varphi_{e,max}$

Shape on selection charts

- Note that $\frac{\rho}{(\varphi_e E)^{1/2}} = \frac{\rho/\varphi_e}{(E/\varphi_e)^{1/2}}$ New material with $\begin{cases} \rho^* = \rho/\varphi_e \\ E^* = E/\varphi_e \end{cases}$



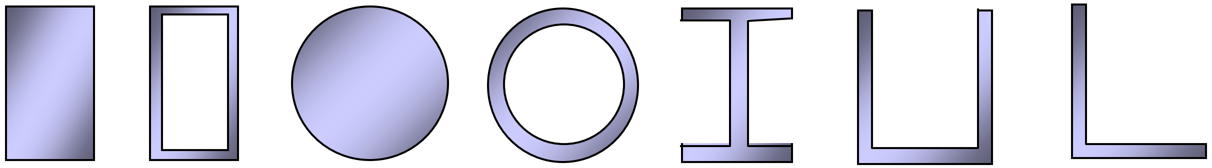
Al: $\varphi_e = 1$

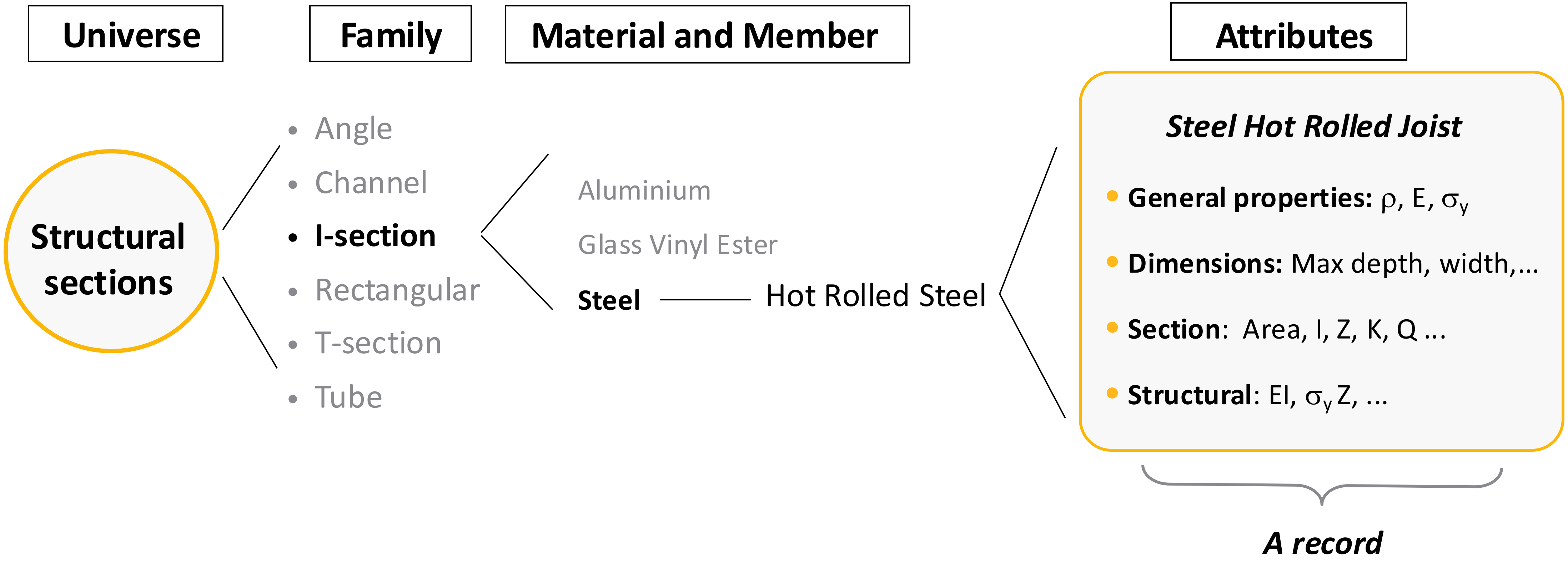
Al: $\varphi_e = 44$

$\frac{\rho}{E^{1/2}} = C$

Data organization: structural sections

Standard prismatic sections





Universe | **Family** | **Material and Member** | **Attributes**

Structural sections

- Angle
- Channel
- **I-section**
- Rectangular
- T-section
- Tube

Aluminium

Glass Vinyl Ester

Steel — Hot Rolled Steel

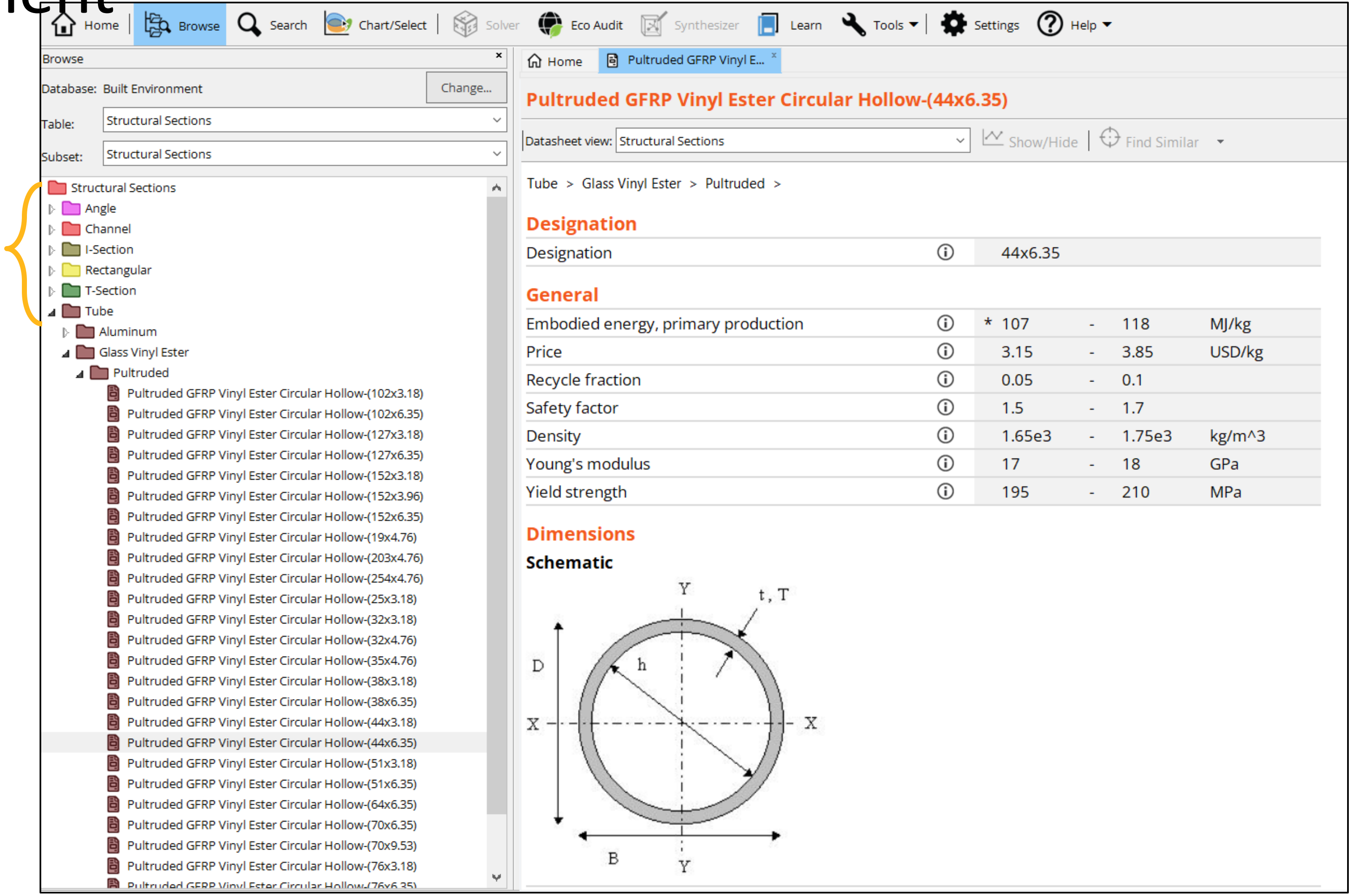
Steel Hot Rolled Joist

- **General properties:** ρ, E, σ_y
- **Dimensions:** Max depth, width,...
- **Section**: Area, I, Z, K, Q ...
- **Structural**: EI, σ_y Z, ...

A record

Ansys Granta EduPack software database for the Built Environment

Records for 1881 sections



Database: Built Environment

Table: Structural Sections

Subset: Structural Sections

- Structural Sections
  - Angle
  - Channel
  - I-Section
  - Rectangular
  - T-Section
  - Tube
    - Aluminum
    - Glass Vinyl Ester
      - Pultruded
        - Pultruded GFRP Vinyl Ester Circular Hollow-(102x3.18)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(102x6.35)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(127x3.18)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(127x6.35)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(152x3.18)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(152x3.96)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(152x6.35)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(19x4.76)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(203x4.76)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(254x4.76)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(25x3.18)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(32x3.18)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(32x4.76)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(35x4.76)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(38x3.18)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(38x6.35)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(44x3.18)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(44x6.35)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(51x3.18)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(51x6.35)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(64x6.35)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(70x6.35)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(70x9.53)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(76x3.18)
        - Pultruded GFRP Vinyl Ester Circular Hollow-(76x6.35)

Pultruded GFRP Vinyl Ester Circular Hollow-(44x6.35)

Datasheet view: Structural Sections

Tube > Glass Vinyl Ester > Pultruded >

Designation

Designation	44x6.35			

General

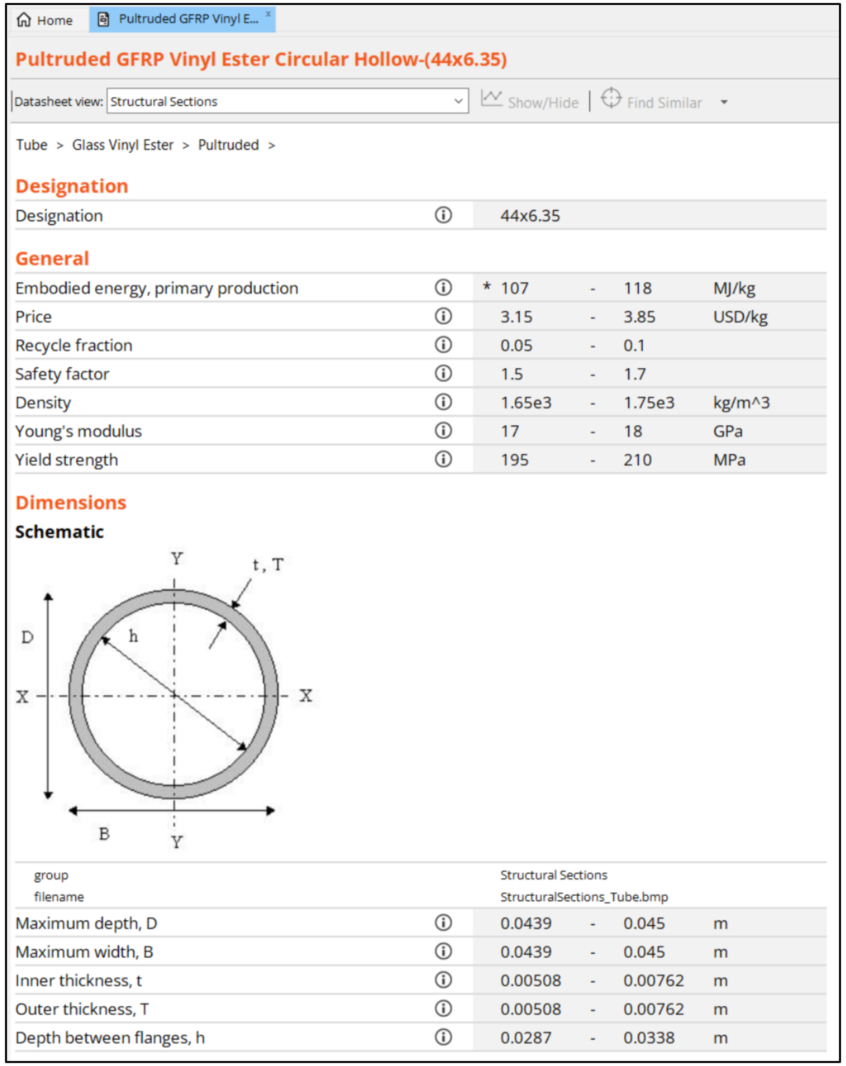
Embodied energy, primary production	* 107	-	118	MJ/kg
Price	3.15	-	3.85	USD/kg
Recycle fraction	0.05	-	0.1	
Safety factor	1.5	-	1.7	
Density	1.65e3	-	1.75e3	kg/m^3
Young's modulus	17	-	18	GPa
Yield strength	195	-	210	MPa

Dimensions

Schematic

Part of a record for a structural section

Pultruded GFRP Vinyl Ester Circular Hollow-(44 x 6.35)



Pultruded GFRP Vinyl Ester Circular Hollow-(44x6.35)

Datasheet view: Structural Sections

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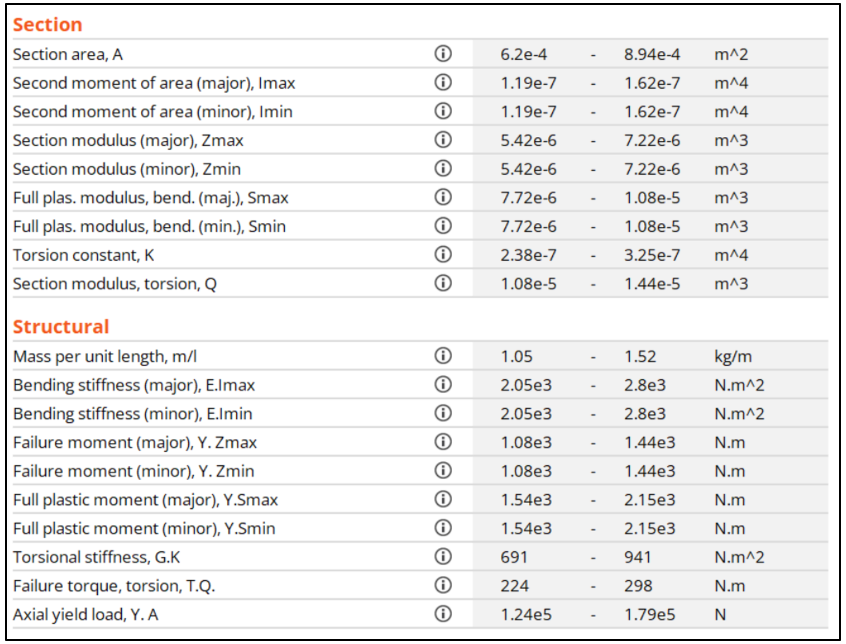
Young's modulus	17	-	18	GPa
Yield strength	195	-	210	MPa

Dimensions

Schematic

group	Structural Sections
filename	StructuralSections_Tube.bmp

Maximum depth, D	0.0439	-	0.045	m
Maximum width, B	0.0439	-	0.045	m
Inner thickness, t	0.00508	-	0.00762	m
Outer thickness, T	0.00508	-	0.00762	m
Depth between flanges, h	0.0287	-	0.0338	m



Section

Section area, A	6.2e-4	-	8.94e-4	m^2
Second moment of area (major), Imax	1.19e-7	-	1.62e-7	m^4
Second moment of area (minor), Imin	1.19e-7	-	1.62e-7	m^4
Section modulus (major), Zmax	5.42e-6	-	7.22e-6	m^3
Section modulus (minor), Zmin	5.42e-6	-	7.22e-6	m^3
Full plas. modulus, bend. (maj.), Smax	7.72e-6	-	1.08e-5	m^3
Full plas. modulus, bend. (min.), Smin	7.72e-6	-	1.08e-5	m^3
Torsion constant, K	2.38e-7	-	3.25e-7	m^4
Section modulus, torsion, Q	1.08e-5	-	1.44e-5	m^3

Structural

Mass per unit length, m/l	1.05	-	1.52	kg/m
Bending stiffness (major), E.Imax	2.05e3	-	2.8e3	N.m^2
Bending stiffness (minor), E.Imin	2.05e3	-	2.8e3	N.m^2
Failure moment (major), Y. Zmax	1.08e3	-	1.44e3	N.m
Failure moment (minor), Y. Zmin	1.08e3	-	1.44e3	N.m
Full plastic moment (major), Y.Smax	1.54e3	-	2.15e3	N.m
Full plastic moment (minor), Y.Smin	1.54e3	-	2.15e3	N.m
Torsional stiffness, G.K	691	-	941	N.m^2
Failure torque, torsion, T.Q.	224	-	298	N.m
Axial yield load, Y. A	1.24e5	-	1.79e5	N

**A selection of attributes are shown*

Example: selection of a beam

Specification

Custom Subset
Structural SubSections

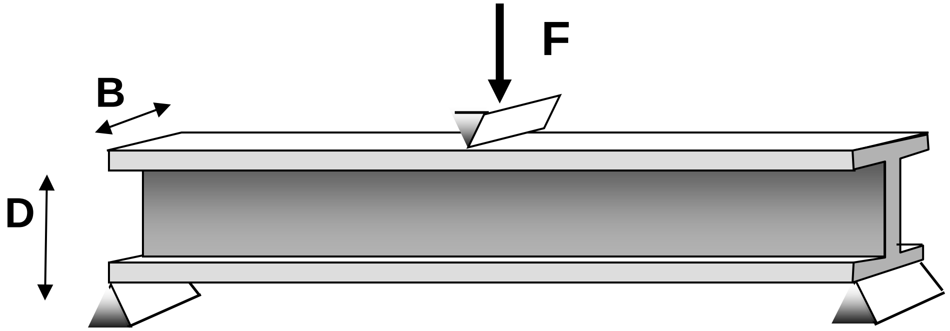
Function ***Beam***

Constraints

Required stiffness:
$E\,I_{max} > 10^5$ **N.m²**

Required strength:
$\sigma_y Z > 10^3$ **N.m**

Dimension
Width B < 150 mm
Depth D < 200 mm



D = beam depth
B = width
I = second moment of area
E = Young's modulus
Z = section modulus
σ_y = yield strength (Y in database)

Applying constraints with a Limit stage

Dimensions	Minimum	Maximum	
Max **depth, D**		0.2	m
Max **width, B**		0.15	m
Structural Attributes			
Bending stiffness E.I$_{max}$	100000		N.m^2
Failure moment Y. Z$_{max}$	1000		N.m

Custom subset of Structural Section

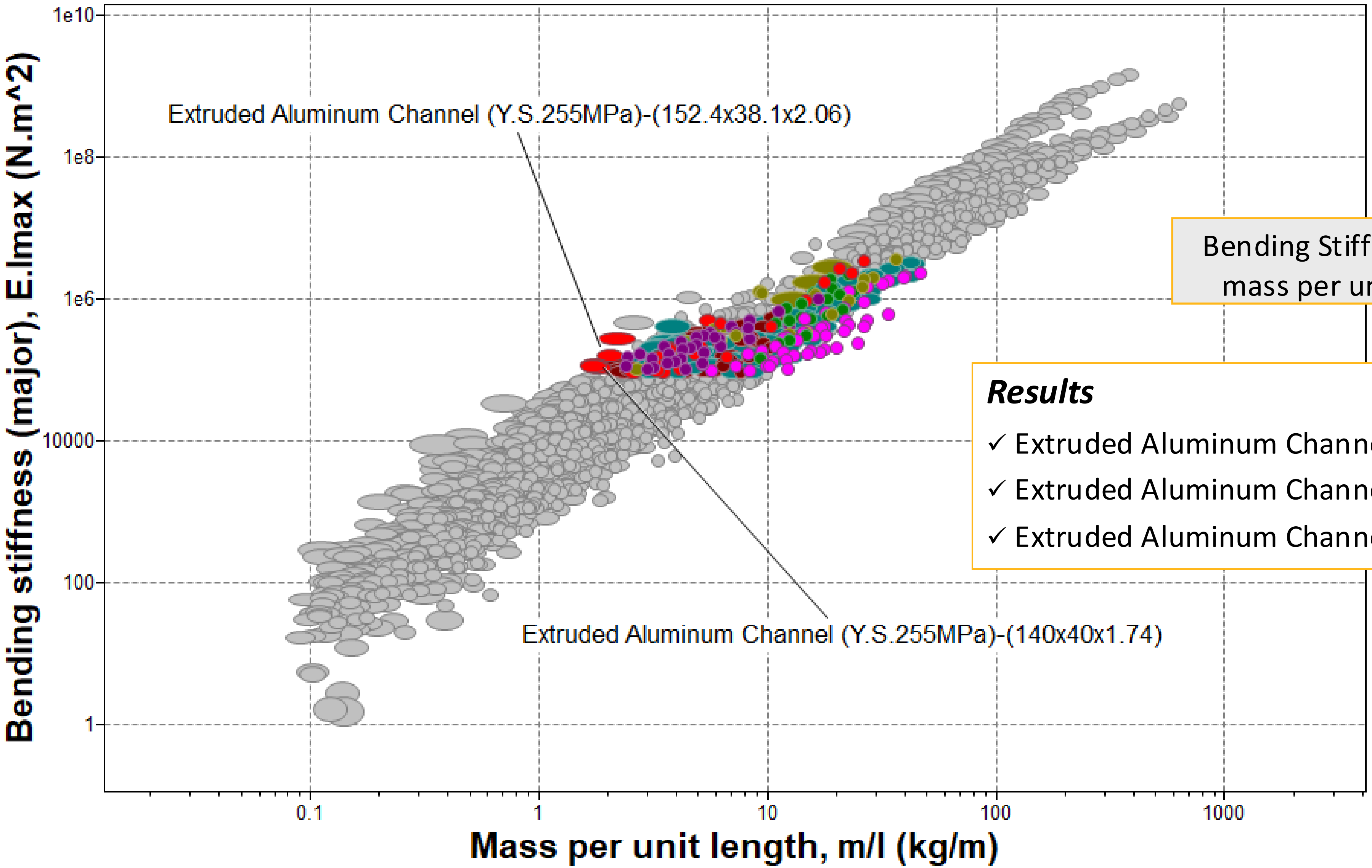
Result : 294 sections out of 1881 meet these constraints

Objectives

- *(a) Find **lightest** beam*
- *(b) Find **cheapest** beam*
- *(c) Find beam with **lowest embodied energy***

That meets the constraints

Minimizing mass for given EI_{max}

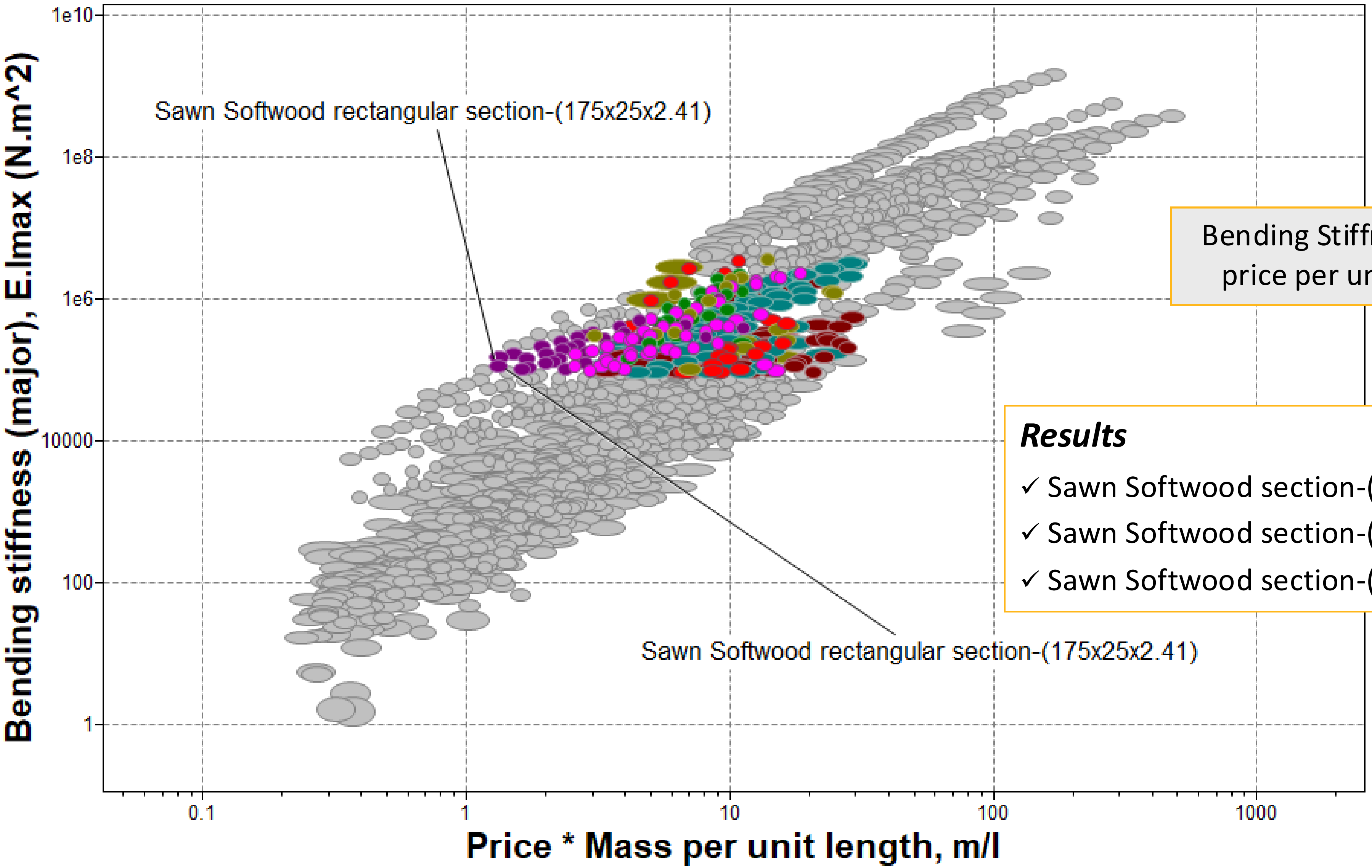


Bending Stiffness EI vs. mass per unit length

Results

- ✓ Extruded Aluminum Channel (140x40x1.74)
- ✓ Extruded Aluminum Channel (152.4x28.6x1.75)
- ✓ Extruded Aluminum Channel (130x50x1.82)

Minimizing cost for given EI_{max}

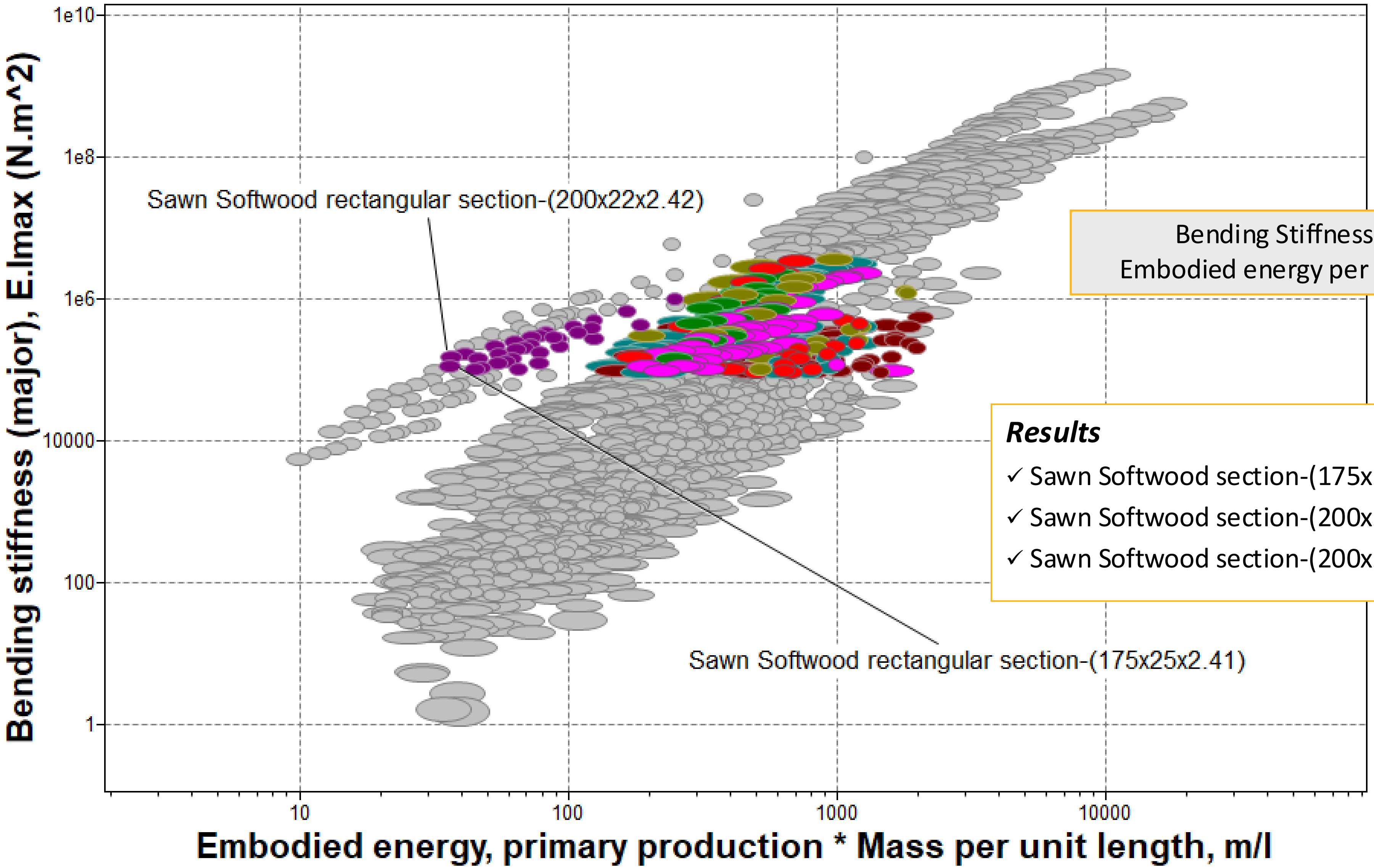


Bending Stiffness EI vs. price per unit length

Results

- ✓ Sawn Softwood section-(175x25x2.41)
- ✓ Sawn Softwood section-(200x22x2.42)
- ✓ Sawn Softwood section-(200x25x2.75)

Minimizing embodied energy for given EI_{max}

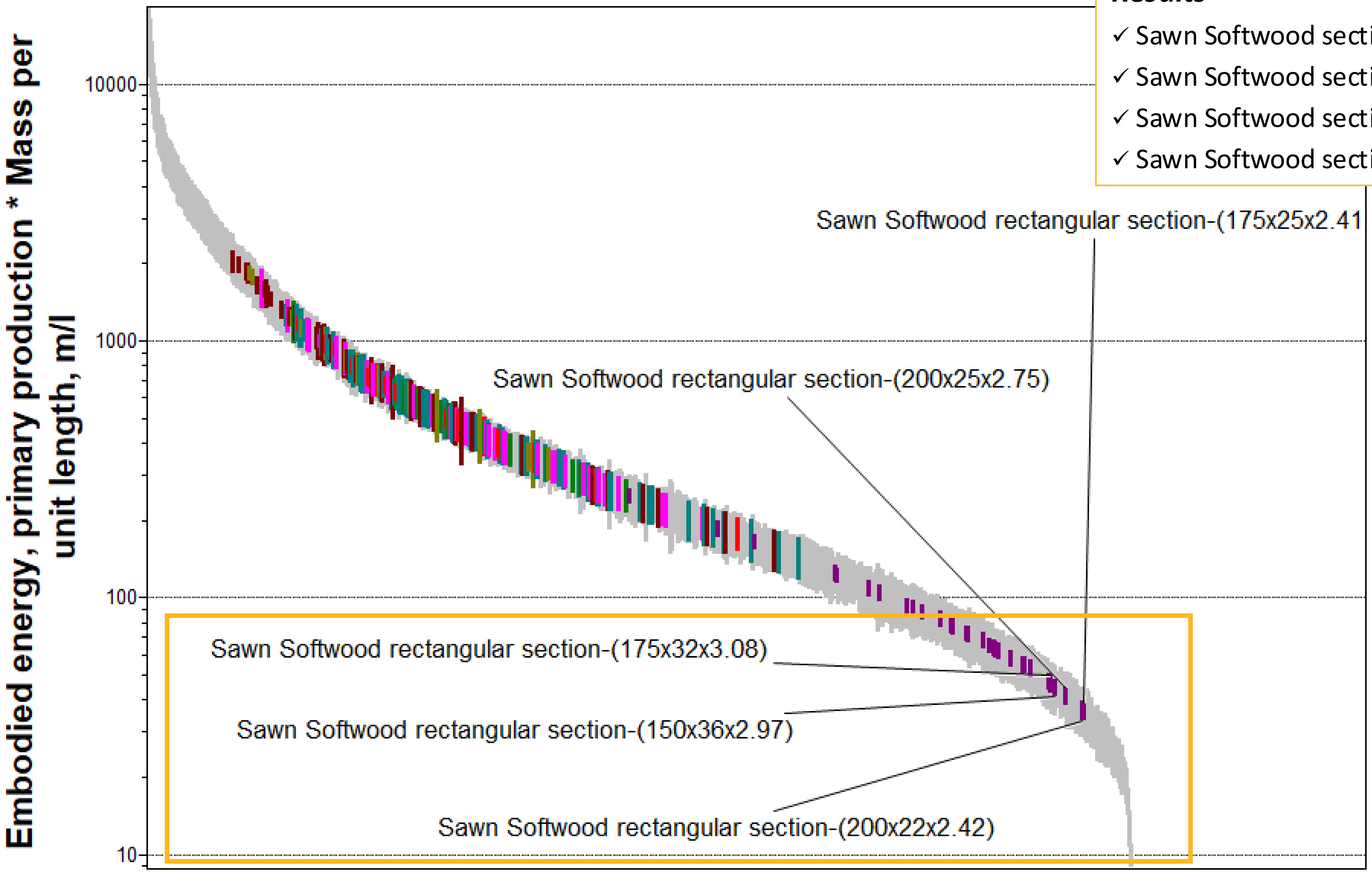


Bending Stiffness EI vs.
Embodied energy per unit length

Results

- ✓ Sawn Softwood section-(175x25x2.41)
- ✓ Sawn Softwood section-(200x22x2.42)
- ✓ Sawn Softwood section-(200x25x2.75)

Minimizing embodied energy for given EI_{max}



Results

- ✓ Sawn Softwood section-(175x25x2.41)
- ✓ Sawn Softwood section-(200x22x2.42)
- ✓ Sawn Softwood section-(200x25x2.75)
- ✓ Sawn Softwood section-(150x36x2.97)

Summary

- When materials carry bending, torsion or axial compression, the section shape becomes important.
- The “shape efficiency” is the amount of material needed to carry the load. It is measured by the shape factor, φ.
- If two materials have the *same* shape, the standard indices for bending (e.g. $\rho / E^{1/2}$) guide the choice.
- If materials can be made -- or are available -- in different shapes, then indices which include the shape (e.g. $\rho / (\varphi E)^{1/2}$) guide the choice.
- The Ansys Granta EduPack software Structural sections data-table and the *Built Environment* database allows standard sections to be explored and selected to meet multiple constraints
- It introduces the idea of choice to optimize weight, price or environmental impact

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